SANTIAGO NUMÉRICO I

Cuarto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales

Facultad de Matemáticas, Pontificia Universidad Católica de Chile, Enero 14 - 16, 2009

A Domain Decomposition Method for Linear Exterior Boundary Value Problems in Elasticity*

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Abstract

In this paper we present new domain decomposition methods for solving linear exterior boundary value problems in elasticity in the plane. Our method is based on the combination of finite element method and Dirichlet–to–Neumann mapping, given in terms of Fourier series, which gives the exact boundary condition on an artificial boundary, to transform the exterior problem into an equivalent mixed boundary value problem in a bounded domain. As a model problem we consider the exterior boundary value problem for the Lamé system. The domain is decomposed into a finite number of subdomains and the Dirichlet data on the interfaces is introduced as the unknown of the associated discrete Steklov–Poincaré problem. Next, we use either a preconditioned Richardson–type method or the preconditioned conjugate gradient method by introducing adjustable Dirichlet–Robin–type preconditioners to solve the problem, which yields iteration–by–subdomains algorithms well suited for parallel computations and they can be naturally implemented on a parallel computing environment. A complete discrete analysis proves that our algorithms have an independent convergence of the stepsize of the mesh.

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*Work partially suported by Fondecyt Chile through the project No. 1070952

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