
SANTIAGO NUMÉRICO I

Cuarto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales

Facultad de Matemáticas, Pontificia Universidad Católica de Chile, Enero 14 - 16, 2009

A stabilized mixed discontinuous Galerkin formulation: A priori and a posteriori error analyses*

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Abstract

In this talk we present an a priori and a posteriori error analysis of a stabilized mixed discontinuous Galerkin formulation for elliptic problems. Our approach requires the introduction of suitable Galerkin least-squares terms (arising from constitutive and equilibrium equations), which allow us to look for the flux unknown in the local Raviart-Thomas space. The unique solvability of the discrete scheme is established avoiding the introduction of lifting operators and allow us to conclude that the rate of convergence of the error, measured in an appropriate norm, is optimal respect to the h -version. Furthermore, we apply Helmholtz decomposition to obtain a reliable and efficient a posteriori error estimate for our approach. Finally, we present several numerical experiments, showing the robustness of the method as well as the theoretical properties of the estimator, thus confirming the capability of the corresponding adaptive algorithms to localize the inner layers, the singularities and/or the large stress regions of the exact solution.

References

- [1] D.N. ARNOLD, F. BREZZI, B. COCKBURN, L.D. MARINI: *Unified analysis of discontinuous Galerkin methods for elliptic problems*. SIAM Journal on Numerical Analysis, vol. 39, 5, pp. 1749-1779, (2001).
- [2] T.P. BARRIOS AND R. BUSTINZA: *An augmented discontinuous Galerkin method for elliptic problems*. Comptes Rendus de l'Academie des Sciences, Series I, vol. 344, pp. 53-58, (2007).

*This research was partially supported by FONDECYT Grants 1080168 and 11060014, by FONDAP and BASAL projects CMM, Universidad de Chile, and by Centro de Investigación en Ingeniería Matemática (CI²MA), Universidad de Concepción.

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- [3] T.P. BARRIOS AND R. BUSTINZA: *A priori and a posteriori error analyses of an augmented Galerkin discontinuous formulation*. IMA Journal of Numerical Analysis, to appear.
- [4] T.P. BARRIOS AND R. BUSTINZA: *An augmented DG scheme for porous media equations*. Numerical Mathematics and Advanced Applications: Proceedings ENUMATH 2007, K. Kunisch, G. Of and O. Steinbach, eds, pp. 315-322, Springer Verlag, 2008.
- [5] T.P. BARRIOS AND R. BUSTINZA: *An a posteriori error analysis of an augmented discontinuous Galerkin formulation for Darcy flow*. Preprint 08-18, Departamento de Ingeniería Matemática, Universidad de Concepción, (2008).
- [6] R. BECKER, P. HANSBO AND M.G. LARSON: *Energy norm a posteriori error estimation for discontinuous Galerkin methods*. Computer Methods in Applied Mechanics and Engineering, vol. 192, pp. 723-733, (2003).