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A new p-interpolation operator for Raviart-Thomas elements and its application to the convergence analysis of the high-order BEM for electro-magnetic scattering*

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Abstract

Interpolation operators (or projectors onto corresponding polynomial spaces) are frequently used in the analysis of discrete methods for time-harmonic Maxwell equations. In particular, a proper choice of these operators is critical for the proof of the discrete compactness property which, together with an appropriate approximability condition, implies the convergence of finite element methods (FEM) for Maxwell eigenvalue problems. When the problem of electro-magnetic scattering (modelled by Maxwell's equations in the exterior domain) is reformulated as a boundary integral equation on the surface of the scatterer, one can apply the boundary element method (BEM) for its approximate solution. The electric field integral equation (EFIE) is one of many possible integral formulations. It is usually discretised by the div-conforming Galerkin BEM based on Raviart-Thomas spaces. Then, as in the FEM for Maxwell's equations, an appropriate interpolation operator is needed to prove convergence of boundary element approximations for the EFIE. However, the existing operators do not easily fit the theoretical framework of the BEM, which is based on negative order Sobolev spaces. This is especially true for high-order methods (p- and hp-BEM).

In this talk we introduce a new $\tilde{\mathbf{H}}^{-1/2}(\mathrm{div})$-conforming p-interpolation operator for Raviart-Thomas elements. This operator has a number of useful properties related to the high-order BEM on piecewise smooth surfaces:

- it assumes sufficiently low $\mathbf{H}^r \cap \tilde{\mathbf{H}}^{-1/2}(\mathrm{div})$-regularity ($r > 0$);
- it commutes with the $\tilde{H}^{-1/2}$-projector;
- it is quasi-stable with respect to polynomial degrees.

Then we apply this interpolation operator to prove convergence of the hp-version of the BEM for the EFIE on piecewise plane (open or closed) surfaces discretised by quasi-uniform meshes.

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