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An entropy inequality for a class of multi-species kinematic flow models with discontinuous flux *

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Abstract

We study a system of conservation laws that models multi-species kinematic flow models with an emphasis on models of multiclass traffic flow [1] and of the creaming of oil-in-water dispersions [6]. The flux is allowed to have a spatial discontinuity which models abrupt changes of road surface conditions or of the cross-sectional area in a settling vessel. For this system, an entropy inequality is proposed that singles out the correct solution at the interface. It is shown that limit solutions generated by a numerical scheme the authors recently proposed [2] satisfy this entropy inequality. It is also shown that limit solutions are entropy admissible, and in the genuinely nonlinear case, satisfy the usual Lax condition for a shock located away from the interface. We present an improvement to our scheme, involving a special interface flux that is activated only at a few grid points where the flux discontinuity is located. Numerical experiments indicate that this interface flux essentially eliminates overshoots that are sometimes present at the interface with our original scheme. We show that the scheme, with or without the interface fix, preserves a natural invariant region.

Related earlier work includes [3, 5, 7]. This contribution is based on work under preparation [4].

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