
SANTIAGO NUMÉRICO I

Cuarto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Facultad de Matemáticas, Pontificia Universidad Católica de Chile, Enero 14 - 16, 2009

Numerical methods for an inverse problem in scalar conservation laws *

RAIMUND BÜRGER [†] ANIBAL CORONEL [‡] MAURICIO SEPÚLVEDA [†]

Abstract

This contribution is concerned with the numerical solution of flux identification problem in scalar conservation laws where the solution in a fixed time is known. This inverse problem is formulated in a variational setting by introducing an objective function which compares, in the L^2 -norm, the simulation and the observation profiles. We consider two numerical methodologies to evaluate the exact gradient of the discrete objective function: the sensitivity equation method and the adjoint equation method. We comment the consequence of shock formation in the differentiability of the cost function in both continuous gradient formulations. In the case of the adjoint sensitivity analysis we interpret the continuous adjoint equation in the sense of reversible solutions and we prove the convergence of the exact gradient to an element of the subdifferential of the cost function. Although, the numerical examples are focus on the context of well-known phenomenological sedimentation model, the identification method can be applied to other one-dimensional hyperbolic models.

References

- [1] BÜRGER A., CORONEL, A., AND SEPULVEDA, M. *A semi-implicit monotone difference scheme for an initial- boundary value problem of a strongly degenerate parabolic equation modelling sedimentation-consolidation processes*. Math. Comp. **75** (2006) 91–112.
- [2] CORONEL, A., JAMES, F. AND SEPULVEDA, M. *Numerical identification of parameters for a model of sedimentation processes*. Inverse Problems **19** (2003) 951–972.

*This research was partially supported by Conicyt through Fondap in Applied Mathematics, Fondecyt 1070694, Fondecyt 1050728 and Fondecyt 11060400.

[†]Departamento de Ingeniería Matemática, Universidad de Concepción, Casilla 160-C, Concepción, Chile.
E-mail: rburger@ing-mat.udec.cl, mauricio@ing-mat.udec.cl

[‡]Departamento de Ciencias Básicas, Facultad de Ciencias, Universidad del Bío-Bío, Casilla 447, Campus Fernando May, Chillán, Chile.
e-mail: acoronel@roble.fdo-may.ubiobio.cl

- [3] JAMES, F. AND SEPÚLVEDA, M. *Convergence results for the flux identification in a scalar conservation law*. SIAM J. Control Optim. **37** (1999) 869–891.
- [4] GLASS, O., *An extension of Oleinik’s inequality for general 1-D scalar conservation laws*, Journal of Hyperbolic Differential Equations, vol. 5, 1, pp. 113-165, (2008)