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Existence, uniqueness and approximation of the Stokes equations in some non-Lipschitz domains *

Ricardo G. Durán † Fernando López García ‡

Abstract

Theoretical and numerical analysis of elliptic problems are strongly based on several results in the theory of Sobolev spaces which are valid for Lipschitz domains (trace theorems, Korn inequality, inf-sup conditions, etc.)

On the other hand, it is known that for some non-Lipschitz domains many of these results are not valid. In this talk we consider a particular class of non-Lipschitz domains, namely, domains with external cusps (for example the complement of two tangent circles or spheres). This kind of domains can be viewed as a particular case of Hölder-α domains.

First we recall our previous works [1, 2] on the analysis and approximation of the Poisson equation in this kind of domains.

Second, we present new results for the Stokes equations. As it is well known, existence, uniqueness and stability of numerical solutions follows from the inf-sup condition and its discrete counterpart. We show by an elementary example that the standard inf-sup condition does not hold for the domains that we are considering. Therefore, a natural question is whether some weaker inf-sup condition can be proved for these domains. To give a positive answer to this question we work with weighted Sobolev norms. We present a result for the general class of Hölder-α domains [3] and a sharper result for particular domains with external cusps.

These generalized inf-sup conditions allow us to apply the general theory of saddle point problems to the analysis of the Stokes equations in these domains.

References

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†Departamento de Matemática, Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires, Argentina, e-mail: rduran@dm.uba.ar

‡Departamento de Matemática, Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires, Argentina, e-mail: flopezg@dm.uba.ar

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