SANTIAGO NUMÉRICO I
Cuarto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Facultad de Matemáticas, Pontificia Universidad Católica de Chile, Enero 14 - 16, 2009

A residual-based a posteriori error estimator for a two-dimensional fluid-solid interaction problem*

Gabriel N. Gatica † George C. Hsiao ‡ Salim Meddahi §

Abstract

In this paper we develop an a posteriori error analysis of a mixed finite element method for a fluid-solid interaction problem posed in the plane. The media are governed by the acoustic and elastodynamic equations in time-harmonic regime, respectively, and the transmission conditions are given by the equilibrium of forces and the equality of the normal displacements of the solid and the fluid. The coupling of primal and dual-mixed finite element methods is applied to compute both the pressure of the scattered wave in the linearized fluid and the elastic vibrations that take place in the elastic body. The finite element subspaces consider continuous piecewise linear elements for the pressure and a Lagrange multiplier defined on the interface, and PEERS for the stress and rotation in the solid domain. We derive a reliable and efficient residual-based a posteriori error estimator for this coupled problem. Suitable auxiliary problems, the continuous inf-sup conditions satisfied by the bilinear forms involved, a discrete Helmholtz decomposition, and the local approximation properties of the Clément interpolant and Raviart-Thomas operator are the main tools for proving the reliability of the estimator. Then, Helmholtz decomposition, inverse inequalities, and the localization technique based on triangle-bubble and edge-bubble functions are employed to show the efficiency.

References

[1] Bernardi, C., Canuto, C. and Maday, Y., *Generalized inf-sup conditions for Chebyshev spectral approximations of the Stokes problem.* SIAM Journal on Numerical Analysis, vol. 25, 6, pp. 1237-1271, (1988).

*This research was partially supported by FONDAP and BASAL projects CMM, Universidad de Chile, and by Centro de Investigación en Ingeniería Matemática (CI^{2}MA), Universidad de Concepción.

†CI^{2}MA and Departamento de Ingeniería Matemática, Facultad de Ciencias Físicas y Matemáticas, Universidad de Concepción, Casilla 160-C, Concepción, Chile, e-mail: ggatica@ing-mat.udec.cl

‡Department of Mathematical Sciences, College of Arts and Sciences, University of Delaware, Newark, DE, USA, e-mail: hsiao@math.udel.edu

§Departamento de Matemáticas, Facultad de Ciencias, Universidad de Oviedo, Calvo Sotelo s/n, Oviedo, España, e-mail: salim@uniovi.es

[2] Gatica, G.N. and Maischak, M., *A posteriori error estimates for the mixed finite element method with Lagrange multipliers*. Numerical Methods for Partial Differential Equations, vol. 21, 3, pp. 421-450, (2005).

[3] Gatica, G.N., Márquez, A. and Meddahi, S., *Analysis of the coupling of primal and dual-mixed finite element methods for a two-dimensional fluid-solid interaction problem*. SIAM Journal on Numerical Analysis, vol. 45, 5, pp. 2072-2097, (2007).

[4] Hsiao, G.C., Kleinman, R.E. and Schuetz, L.S., *On variational formulations of boundary value problems for fluid-solid interactions*. In: Elastic Wave Propagation, M.F. McCarthy and M.A. Hayes (Editors), pp. 321-326, Elsevier Science Publishers B.V. (North-Holland), 1989.

[5] Meddahi, S. and Sayas F.-J., *Analysis of a new BEM-FEM coupling for two dimensional fluid-solid interaction*. Numerical Methods for Partial Differential Equations, vol. 21, 6, pp. 1017-1042, (2005).

[6] Verfürth, R., A Review of A Posteriori Error Estimation and Adaptive Mesh-Refinement Techniques. Wiley-Teubner (Chichester), 1996.