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Numerical Simulation Of Two-Dimensional Bingham Fluid Flow By Semismooth Newton Methods

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Abstract

This work is devoted to the numerical simulation of two-dimensional stationary Bingham fluid flow by semi-smooth Newton methods. Bingham fluids are visco-plastic materials which behave like incompressible fluids in the regions where the stress reaches a given yield and like solids in the regions where the stress remains below that threshold. The mathematical models for such materials involve the constituent law for viscous incompressible fluids, with an extra stress tensor component modeling the visco-plastic effects.

We are concerned with Bingham fluid flow in a given domain $\Omega \subset \mathbb{R}^2$, considering non-homogeneous Dirichlet and stress-free boundary conditions. We analyze the modeling elliptic variational inequality of the second kind as an equivalent minimization problem and, using Fenchel's duality, we obtain an optimality system which characterizes the primal and dual solutions. Since the solution to the dual problem is not unique, a family of Tikhonov regularized problems is introduced and the convergence of the regularized solutions to the original one is studied.

For the discretization of each regularized optimality system, a finite element method with (cross-grid $\mathbb{P}_1$) - $\mathbb{Q}_0$ elements is utilized. The chosen pair is known to satisfy the Ladyzhenskaya - Babuska - Brezzi condition and allows also to obtain a direct relation between the discrete primal and dual variables.

For the solution of the resulting system of nonsmooth equations, we propose a semismooth Newton algorithm. Using an additional relaxation of the incompressibility condition a modified reduced system matrix is constructed and a descent direction is obtained from each semismooth Newton iteration. The local superlinear convergence of the method is also proved. Finally, several numerical experiments are carried out in order to investigate the behavior and efficiency of the method.

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