SANTIAGO NUMÉRICO I
Cuarto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Facultad de Matemáticas, Pontificia Universidad Católica de Chile, Enero 14 - 16, 2009

Hybrid boundary elements scheme for modeling flat structures in $\mathbb{R}^3$

Carlos Jerez-Hanckes * Jean-Claude Nédélec †

Abstract

We present an augmented boundary element method for modeling elliptic and wave propagation problems in $\mathbb{R}^3$ with Dirichlet conditions imposed over flat surfaces having very large aspect ratios. For this, we study unbounded connected domains $\Omega := \mathbb{R}^d \setminus \bar{\Gamma}_m$, $d = 2, 3$, where Γ_m is an orientable manifold of co-dimension one, e.g., a line segment or a plane. Thus, Ω is not even Lipschitz and problems defined therein usually fall in the category of screen, crack or interface problems [1], [2], [3], for which solutions are known to possess singular behaviors [4] and classical Galerkin or collocation methods show poor convergence.

The talk is organized as follows. We first analyze simple problems in $\mathbb{R}^2$ with Γ_m described by a Jordan curve, and observe that the associated single layer potentials can be reduced to compactly perturbed logarithmic integral operators. Their solutions are shown to be accurately given by weighted Tchebychev polynomials. Then, we extend these ideas to manifolds in $\mathbb{R}^3$ with only one bounded direction, e.g., infinite strips or cylinders, and show that localized single layer operators also portray logarithmic singularities. Entirely bounded surfaces with large length-to-width ratios are next considered. If the manifold boundary $\partial\Gamma_m$ is Lipschitz, corner singularities may show up. To handle this, our numerical scheme uses the previous observations and employs different discretization bases according to the encountered singular behavior – corner or edge. As a concrete application, we show results for electrostatic and elastic wave generation problems for the so-called surface acoustic waves interdigital transducers (SAW IDTs) [5].

References

[1] E. Stephan. *Boundary integral equations for screen problems in* $\mathbb{R}^3$. Integral Equations Operator Theory, vol. 10, 2, pp. 236-257, (1987).

*Seminar für Angewandte Mathematik, ETH Zürich, Switzerland, e-mail: cjerez@math.ethz.ch
†Centre de Mathématiques Appliquées, École Polytechnique, 91128 Palaiseau, France, e-mail: nedelec@cmap.polytechnique.fr

[2] M. Costabel and M. Dauge. *Crack singularities for general elliptic systems.* Mathematische Nachrichten, vol. 235, 1, pp. 29-49, (2002).

[3] S. Nicaise and A.-M. Sändig. *General interface problems* I. Mathematical Methods in Applied Sciences, vol. 17, 6, pp. 395-429, (1994).

[4] P. Grisvard. *Elliptic Problems in Nonsmooth Domains.* Pitman, Ireland, 1985.

[5] C. Jerez-Hanckes, V. Laude, J.-C. Nédelec and R. Lardat. *3-D Electrostatic Hybrid Elements Model for SAW Interdigital Transducers.* IEEE Ultrasonics, Ferroelectrics and Frequency Control Transactions vol. 55, 3, pp. 686–695, (2008).