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Estimates for Raviart-Thomas and Nédélec Elements on Anisotropic Meshes

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Abstract

In this talk we consider estimates for the Raviart-Thomas [6, 7] and Nédélec [4] interpolation operators of any order on tetrahedral meshes with arbitrarily narrow elements. More precisely, we obtain interpolation error estimates on meshes satisfying two different geometrical restrictions, namely, the "regular vertex property" (RVP) and the "maximum angle condition" (MAC) [1]. These two conditions allow for meshes that not satisfy the standard shape regularity assumption [2], which appear naturally, for instance, in the approximations of boundary layers. The RVP is a stronger condition than the MAC.

The estimates are obtained in each element of the mesh paying attention to the dependence of the constants on the geometrical properties of the element. The global estimate is obtained adding the individual ones. Then we are interested in two kind of estimates: (1) estimates valid uniformly for all elements having a particular geometrical property, and (2) anisotropic estimates also valid uniformly for some class of elements. We say that an estimate is of anisotropic type when in front each derivative appear the lengths of the element in the corresponding directions.

Related results were previously obtained, for instance, in [3] for Raviart-Thomas interpolation in two dimensions or in three dimensions under the RVP, and in [5] for the Nédélec interpolation of lowest degree.

For the Raviart-Thomas interpolation we obtain error estimates valid uniformly under the MAC, but anisotropic estimates can be proved only under the RVP. This is not the case for the Nédélec interpolation, for which we can obtain anisotropic error estimates also under the MAC.

These results are partly joint work with Thomas Apel, Gabriel Acosta, and Ricardo G. Durán.

References

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