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A coupled mixed finite element method for the interaction problem between electromagnetic field and elastic body*

Gabriel N. Gatica † George C. Hsiao ‡ Salim Meddahi §

Abstract

This paper deals with the coupled problem arising from the interaction of a time harmonic electromagnetic field with a three-dimensional elastic body. More precisely, we consider a suitable transmision problem holding between the solid and a sufficiently large annular region surrounding it, and aim to compute both the magnetic component of the scattered wave and the stresses that take place in the obstacle. To this end, we assume Voigt's model, which allows interaction only through the boundary of the body, and employ a dual-mixed variational formulation in the solid media. As a consequence, one of the two transmission conditions becomes essential, whence it is enforced weakly through the introduction of a Lagrange multiplier. The abstract framework developed in a recent work by A. Buffa is applied next to show that our coupled variational formulation is well posed. In addition, we define the corresponding Galerkin scheme by using PEERS in the solid and the edge finite elements of Nédélec in the electromagnetic region. Then, we prove that the resulting coupled mixed finite element scheme is uniquely solvable and convergent. Moreover, optimal a priori error estimates are derived in the usual way. Finally, some numerical results illustrating the analysis and the good performance of the method are also reported.

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†CI^{2}MA and Departamento de Ingeniería Matemática, Facultad de Ciencias Físicas y Matemáticas, Universidad de Concepción, Casilla 160-C, Concepción, Chile, e-mail: ggatica@ing-mat.udec.cl

‡Department of Mathematical Sciences, College of Arts and Sciences, University of Delaware, Newark, DE, USA, e-mail: hsiao@math.udel.edu

§Departamento de Matemáticas, Facultad de Ciencias, Universidad de Oviedo, Calvo Sotelo s/n, Oviedo, España, e-mail: salim@uniovi.es

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