SANTIAGO NUMÉRICO I
Cuarto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Facultad de Matemáticas, Pontificia Universidad Católica de Chile, Enero 14 - 16, 2009

A Parameter–Uniform Finite Difference Method for a Singularly Perturbed Multiscale Linear Dynamical System*

S. Valarmathi † John J.H. Miller ‡

Abstract

A system of singularly perturbed ordinary differential equations of first order with given initial conditions is considered. The leading term of each equation is multiplied by a small positive parameter, which can be arbitrarily small. These parameters are, in general, unequal. The components of the solution exhibit overlapping layers corresponding to the various distinct time scales occurring in the solution of the problem. It is well-known that standard numerical methods do not perform in a robust way, when they are used to solve a singularly perturbed problem of this kind. In this talk a new numerical method is constructed. First a Shishkin piecewise–uniform mesh is introduced, which is used, in conjunction with a classical finite difference discretisation, to form the new numerical method for solving this problem. It is then proved that the numerical approximations obtained from this method are essentially first order convergent uniformly with respect to all of the parameters. Extensive numerical computations are presented to illustrate the utility of this new method in practice.

References

[1] P.A. Farrell, A. Hegarty, J. J. H. Miller, E. O'Riordan, G. I. Shishkin, *Robust Computational Techniques for Boundary Layers*, Applied Mathematics & Mathematical Computation (Eds. R. J. Knops & K. W. Morton), Chapman & Hall/CRC Press (2000).

[2] S.Hemavathi, S. Valarmathi, *A parameter uniform numerical method for a system of singularly perturbed ordinary differential equations*, Proceedings of the International Conference on Boundary and Interior Layers, BAIL 2006, Goettingen (2006).

*This research was supported by INCA. The first author was also supported by the UGC, New Delhi, India under the Minor Research Project-X Plan period

†Department of Mathematics, Bishop Heber College(Autonomous), Tiruchirappalli-620 017, Tamil Nadu, India, e-mail: valarmathi07@gmail.com

‡Institute for Numerical Computation and Analysis, Dublin, Ireland, e-mail: jm@incaireland.org

[3] J. J. H. Miller, E. O'Riordan, G.I. Shishkin, *Fitted Numerical Methods for Singular Perturbation Problems*, World Scientific Publishing Co., Singapore, New Jersey, London, Hong Kong (1996).