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A residual-based a posteriori error estimator for a fully mixed formulation of the Stokes-Darcy coupled problem. *

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Abstract

In this paper we develop an a posteriori error analysis of a fully mixed finite element method for the coupling of fluid flow with porous media flow. The flows are governed by the Stokes and Darcy equations, respectively, and the transmission conditions are given by mass conservation, balance of normal forces, and the Beavers-Joseph-Saffman law. The finite element subspaces consider Raviart-Thomas elements for the stress tensor of the Stokes equations, piecewise constants and Raviart-Thomas elements for the velocities, piecewise constants for the pressure in the porous medium, and continuous piecewise linear elements for the Lagrange multipliers defined on the interface. We derive a reliable and efficient residual-based a posteriori error estimator for this coupled problem. The proof of reliability makes use of Helmholtz decompositions and local approximation properties of the Clément interpolant and Raviart-Thomas operator. On the other hand, the localization technique based on triangle-bubble and edge-bubble functions constitute the main tools for proving the efficiency of the estimator. Finally, some numerical results illustrating the analysis and confirming the good performance of the corresponding adaptive algorithm are reported.

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