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Convergence of a finite volume method for cardiac propagation*

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Abstract

A finite volume method for solving the monodomain and bidomain models for the electrical activity of myocardial tissue is presented. These models consist of a parabolic PDE and a system of a degenerate parabolic and an elliptic PDE, respectively, for certain electric potentials, coupled to an ODE for the gating variable. Existence and uniqueness of the approximate solution is proved. It is also shown that the scheme converges to the corresponding weak solution for the monodomain model, and also for the bidomain equations in the special case of fibers aligned with the axis. Numerical examples in two and three space dimensions indicate experimental rates of convergence slightly above first order for both models. In addition, since typical solutions of the studied models exhibit wavefronts with steep gradients, the finite volume scheme is enriched by a fully adaptive multiresolution method, whose basic purpose is to concentrate computational effort on zones of strong variation of the solution. Time adaptivity is achieved by two alternative devices, namely locally varying time stepping and a Runge-Kutta-Fehlberg-type adaptive time integration. Finally, the optimal choice of the threshold for discarding non-significant information in the multiresolution representation of the solution is addressed.

References

[1] M. Bendahmane, R. Bürger and R. Ruiz, *Convergence of a finite volume scheme for cardiac propagation.* Submitted.

[2] M. Bendahmane, R. Bürger and R. Ruiz, *A multiresolution space-time adaptive scheme for the bidomain model in electrocardiology.* Preprint 2008-10, Depto. Ing. Mat., U. de Concepción, available from `http://www.ing-mat.udec.cl/`; submitted.

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[3] M. Bendahmane and K.H. Karlsen, *Analysis of a class of degenerate reaction-diffusion systems and the bidomain model of cardiac tissue.* Netw. Heterog. Media 1 (2006) 185–218.

[4] Y. Bourgault, Y. Coudière and C. Pierre, *Existence and uniqueness of the solution for the bidomain model used in cardiac electro-physiology.* Nonlin. Anal. Real World Appl. 10 (2009) 458–482.

[5] R. Eymard, Th. Gallouët and R. Herbin, *Finite Volume Methods.* In: P.G. Ciarlet, J.L. Lions (eds.), Handbook of Numerical Analysis, vol. VII, North-Holland, Amsterdam, 713–1020 (2000).

[6] S. Müller and Y. Stiriba, *Fully adaptive multiscale schemes for conservation laws employing locally varying time stepping.* J. Sci. Comput. 30 (2007) 493–531.