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Error estimates for the finite volume discretization for the porous medium equation*

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Abstract

This work is motivated by a combined mixed finite element (MFE) - finite volume (FV) scheme of a two phase flow model for the heap leaching of copper ores modeled by a degenerate parabolic equation

$$\partial_t u - \nabla \cdot (\nabla \beta(u) + F(u)) = r(u), \quad \text{in } Q_T \equiv (0,T) \times \Omega.$$

Initially we have $u(0) = u^0$ in Ω, whereas $u = 0$ on $\partial\Omega$. In the above $0 < T < \infty$ is fixed, Ω is a bounded domain in $\mathbb{R}^d (d \geq 1)$ with a Lipschitz continuous boundary. The function $\beta : \mathbb{R} \to \mathbb{R}$ is non-decreasing and differentiable. By degeneracy we mean a vanishing diffusion, namely $\beta'(u) = 0$ for some u. We prove error estimates for the finite volume discretization for this model. Several numerical results illustrating the performance of the algorithm are provided.

References

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