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Finite Element Error Analysis for State-Constrained Optimal Control of the Stokes Equations

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Abstract

An optimal control problem for 2d and 3d Stokes equations is investigated with pointwise inequality constraints on the state and the control. The paper is concerned with the full discretization of the control problem allowing for different types of discretization of both the control and the state. For instance, piecewise linear and continuous approximations of the control are included in the present theory. Under certain assumptions on the L^∞ -error of the finite element discretization of the state, error estimates for the control are derived which can be seen to be optimal since their order of convergence coincides with the one of the interpolation error. The assumptions of the L^∞ -finite-element-error can be verified for different numerical settings. Finally the results of two numerical experiments are presented.

References

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