SANTIAGO NUMÉRICO I
Cuarto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales

Facultad de Matemáticas, Pontificia Universidad Católica de Chile
Santiago, Enero 14 - 16, 2009

PROGRAMA y RESÚMENES

Índice

1. Introducción

El **Cuarto Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales** ha sido organizado en conferencias secuenciales de **45 y 30 minutos** de duración (40 y 25 minutos de exposición, respectivamente, y 5 minutos para preguntas y comentarios). Todas las charlas se llevarán a cabo en el Auditorio Ninoslav Bralić de la Facultad de Matemáticas.

En las páginas siguientes se detalla primero la programación correspondiente y luego se incluyen los resúmenes de cada uno de los trabajos (en orden alfabético, según autores). Cuando hay más de un autor, aquel que aparece subrayado corresponde al expositor.

Los organizadores expresamos nuestro agradecimiento a los auspiciadores que se indican a continuación, los cuales han aportado gran parte de los recursos necesarios para el financiamiento de este evento:

- Centro de Modelamiento Matemático (CMM) de la Universidad de Chile,
- Departamento de Ingeniería Matemática de la Universidad de Concepción, y
- Facultad de Matemáticas de la Pontificia Universidad Católica de Chile.

Igualmente, extendemos nuestro reconocimiento y gratitud a todos los expositores, quienes han hecho posible la realización de **Santiago Numérico I**.

Comité Organizador

Santiago, Enero de 2009

2. Miércoles, 14 de Enero

8.30-9.15 INSCRIPCIÓN

9.15-9.30 BIENVENIDA DEL DECANO

[*Moderador*: N. HEUER]

9.30-10.15 L.F. Demkowicz: *hp-adaptive finite elements for multiphysics wave propagation problems.*

10.15-10.45 T. Tran, D. Pham: *Efficient meshless methods for pseudodifferential equations on the sphere.*

10.45-11.15 COFFEE BREAK

11.15-11.45 G.N. Gatica, G.C. Hsiao, S. Meddahi: *A coupled mixed finite element method for the interaction problem between electromagnetic field and elastic body.*

11.45-12.15 M. Bendahmane, R. Bürger, R. Ruiz: *Convergence of a finite volume method for cardiac propagation.*

12.15-12.45 F.-J. Sayas: *BEM-FEM coupling: back to the beginning.*

12.45-15.00 ALMUERZO

[*Moderador*: S. MEDDAHI]

15.00-15.30 R. Bürger, K.H. Karlsen, J.D. Towers: *An entropy inequality for a class of multi-species kinematic flow models with discontinuous flux.*

15.30-16.00 T.P. Barrios, J.M. Cascón, G.C. García: *An a posteriori error analysis for the stream function and vorticity formulation of the Stokes problem.*

16.00-16.30 A.L. Lombardi: *Estimates for Raviart-Thomas and Nédélec elements on anisotropic meshes.*

16.30-17.00 COFFEE BREAK

17.00-17.30 A. Bespalov, N. Heuer: *A new p-interpolation operator for Raviart-Thomas elements and its application to the convergence analysis of the high order BEM for electro-magnetic scattering.*

17.30-18.00 J.C. de los Reyes, S. González: *Numerical simulation of two-dimensional Bingham fluid flow by semismooth Newton methods.*

18.00-18.30 G.N. Gatica, G.C. Hsiao, S. Meddahi: *A residual-based a posteriori error estimator for a two-dimensional fluid-solid interaction problem.*

19.30 COCKTAIL DE BIENVENIDA

3. Jueves, 15 de Enero

[*Moderador*: F.-J. SAYAS]

9.30-10.15 R. Durán, F. López-García: *Existence, uniqueness and approximation of the Stokes equations in some non-Lipschitz domains.*

10.15-10.45 R.A. Araya, P. Venegas: *An a posteriori error estimator for a unsteady advection-diffusion equation.*

10.45-11.15 COFFEE BREAK

11.15-11.45 J.C. de los Reyes, C. Meyer, B. Vexler: *Finite element error analysis for state-constrained optimal control of the Stokes equations.*

11.45-12.15 M. Durán, I. Muga, J.-C. Nédélec: *A radiation condition for time-harmonic elastic waves in half-spaces with free boundary.*

12.15-12.45 I.S. Pop, M. Sepúlveda: *Error estimates for the finite volume discretization of the porous medium equation.*

12.45-15.00 ALMUERZO

[*Moderador*: R. BÜRGER]

15.00-15.30 M. Maischak: *Mixed FEM-BEM coupling for nonlinear transmission problems with Signorini contact.*

15.30-16.00 R. Bürger, A. Coronel, M. Sepúlveda: *Numerical methods for an inverse problem in scalar conservation laws.*

16.00-16.30 M. de Buhan, P. Frey: *Modelling and simulation of the viscoelastic behavior of brain structures: preliminary results.*

16.30-17.00 COFFEE BREAK

17.00-17.30 M. Astorino, F. Chouly, M.A. Fernández: *An added-mass free semi-implicit coupling scheme for fluid-structure interaction.*

17.30-18.00 M.A. Barrientos, M.E. Mellado: *A domain decomposition method for linear exterior boundary value problems in elasticity.*

18.00-18.30 E. Hernández, E. Otárola: *Superconvergence scheme of a locking free FEM in a Timoshenko optimal control problem.*

20.00 CENA DE CAMARADERÍA (**QUINCHO**)

4. Viernes, 16 de Enero

[*Moderador*: R. ARAYA]

9.30-10.15 P. Jimack: *Moving mesh finite element methods for the adaptive solution of transient PDEs with moving boundaries.*

10.15-10.45 G.R. Barrenechea, L.P. Franca, C. Harder, F. Valentin: *Pressure projection methods arising from an enriched finite element approach.*

10.45-11.15 COFFEE BREAK

11.15-11.45 T.P. Barrios, R. Bustinza: *A stabilized mixed discontinuous Galerkin formulation: a priori and a posteriori error analyses.*

11.45-12.15 S. Gutiérrez, J. Mura: *About computing the nonlinear interaction between weakly converging sequences and its influence in optimal design and nonlinear elasticity.*

12.15-12.45 R. Durán, R. Rodríguez, F. Sanhueza: *Computation of the vibration modes of a Reissner-Mindlin laminated plate.*

12.45-15.00 ALMUERZO

[*Moderador*: G. GATICA]

15.00-15.30 S. Valarmathi, J.J.H. Miller: *A parameter-uniform finite difference method for a singularly perturbed multiscale linear dynamical system.*

15.30-16.00 C. Jerez-Hanckes, J.-C. Nédélec: *Hybrid boundary elements scheme for modeling flat surfaces in* $\mathbb{R}^3$.

16.00-16.30 E.G. Reyes: *Explicit solutions to nonlinear partial differential equations via nonlocal symmetries.*

16.30-17.00 COFFEE BREAK

17.00-17.30 G.N. Gatica, R.E. Oyarzúa, F.-J. Sayas: *A residual-based a posteriori error estimator for a fully mixed formulation of the Stokes-Darcy coupled problem.*

17.30-18.00 M. Durán, M. Maturana, J.-C. Nédélec, S. Ossandón: *On the calculation of Maxwell's eigenfrequencies using integral equations for a buried landmine.*

18.00-18.30 M. Healey, N. Heuer: *Mortar boundary elements.*

18.30 CIERRE

5. Resúmenes

An a posteriori error estimator for a unsteady advection-diffusion equation*

Rodolfo Araya† Pablo Venegas‡

Abstract

In this work we introduce an a posteriori error estimates for the unsteady advection-diffusion-reaction equation in two space dimensions. For the discretization we use backward Euler in time, and continuous, piecewise linear triangular finite elements in space together with a stabilized scheme. The error is bounded above and below by an explicit error estimator based on the residual. Numerical results are presented for uniform triangulations and constant time steps. The quality of our error estimator is discussed. An adaptive algorithm is then proposed and numerical results demonstrate the efficiency of our approach.

References

[1] Bergam, A., Bernardi, C. and Mghazli, Z., *A posteriori analysis of the finite element discretization of some parabolic equations.* Math. Comp., vol. 74, 251, pp. 1117–1138, (2004).

[2] Picasso, M., *An adaptive finite element algorithm for a two-dimensional parabolic problem.* Comput. Meth. Appl. Mech. Eng.,vol. 167, pp. 223–237, (1998).

[3] Verfürth, R., *A posteriori error estimates for nonlinear problems: $L^r(0,T;W^{1,\rho}(\Omega))$-error estimates for finite element discretizations of parabolic equations.* Numer. Meth. for PDE, vol. 14, pp. 487–518, (1998).

*This research was partially supported by FONDAP and BASAL projects CMM, Universidad de Chile, and by Centro de Investigación en Ingeniería Matemática (CI^{2}MA), Universidad de Concepción.

†CI^{2}MA and Departamento de Ingeniería Matemática, Facultad de Ciencias Físicas y Matemáticas, Universidad de Concepción, Casilla 160-C, Concepción, Chile, e-mail: `raraya@ing-mat.udec.cl`

‡Departamento de Ingeniería Matemática, Facultad de Ciencias Físicas y Matemáticas, Universidad de Concepción, Casilla 160-C, Concepción, Chile, e-mail: `pvenegas@ing-mat.udec.cl`

An added-mass free semi-implicit coupling scheme for fluid-structure interaction *

Matteo Astorino † Franz Chouly ‡ Miguel A. Fernández §

Abstract

We propose a semi-implicit coupling scheme for the numerical simulation of fluid-structure interaction systems involving a viscous incompressible fluid. The scheme is stable irrespectively of the so-called added-mass effect (fluid and solid densities which are close and/or domain which is slender) [5]. Moreover, it allows for conservative time-stepping within the structure. The efficiency of the scheme is based on the explicit splitting of the viscous effects and geometrical/convective non-linearities, through the use of the Chorin-Temam projection scheme within the fluid [6]. Stability relies on the implicit treatment of the pressure stresses and on the Nitsche's treatment of the viscous coupling [2,3,4]. The numerical stability of the scheme is proved theoretically through *a priori* energy estimation [1]. Numerical results in two and three dimensions illustrate the stability and efficiency of the scheme as well as its potentiality in the context of blood flow simulations [7].

References

[1] Astorino M., Chouly F. and Fernández M.A., *An added-mass free semi-implicit coupling scheme for fluid-structure interaction.* C. R. Math. Acad. Sci. Paris, In Review (2008).

[2] Becker R., Hansbo P. and Stenberg R., *A finite element method for domain decomposition with non-matching grids.* M2AN Math. Model. Numer. Anal., vol. 37, pp. 209-225 (2003).

*This research was partially supported by INRIA through the CardioSense3D project and by the ANR agency through the PITAC project.

†INRIA, REO team, Rocquencourt BP 105, F–78153 Le Chesnay Cedex, France,
e-mail: matteo.astorino@inria.fr

‡INRIA, REO team, Rocquencourt BP 105, F–78153 Le Chesnay Cedex, France,
e-mail: franz.chouly@inria.fr

§INRIA, REO team, Rocquencourt BP 105, F–78153 Le Chesnay Cedex, France,
e-mail: miguel.fernandez@inria.fr

[3] Burman E. and Fernández M.A., *Stabilized explicit coupling for fluid-structure interaction using Nitsche's method*, C. R. Math. Acad. Sci. Paris, vol. 345, pp. 467-472 (2007).

[4] Burman E. and Fernández M.A., *Stabilization of explicit coupling in fluid-structure interaction involving fluid incompressibility*, INRIA Research Report RR-6445 (2008).

[5] Causin P., Gerbeau J.-F. and Nobile F., *Added-mass effect in the design of partitioned algorithms for fluid-structure problems.* Comput. Methods Appl. Mech. Engrg., vol. 194, pp. 4506-4527 (2005).

[6] Fernández M.A., Gerbeau J.F. and Grandmont C., *A projection semi-implicit scheme for the coupling of an elastic structure with an incompressible fluid*, Int. J. Num. Meth. Engrg., vol. 69, pp. 794-821 (2007).

[7] Quarteroni A., Formaggia, L. and Veneziani A. (editors), *Cardiovascular Mathematics*, Ch. 9. Algorithms for fluid-structure interaction problems. Springer Verlag. In preparation.

Pressure projection methods arising from an enriched finite element approach*

Gabriel R. Barrenechea † Leopoldo P. Franca ‡
Christopher Harder § Frederic Valentin ¶

Abstract

To make some of the simplest and desirable pair of spaces inf-sup stable for the Stokes and the Darcy models, namely the P_1/P_0, P_1/P_1 and P_1/P_1^{dis} elements, this work proposes a Petrov-Galerkin strategy relied on velocity and pressure enhanced spaces (see [1, 2, 3, 4] for related results). The enriching functions turn out to be the solutions of local mixed problems at element level driven by residuals and spurious modes with degree of freedom fixed by the original pair of elements. Having incorporates the element wise contribution the now stable methods recover some of the pressure projection methods recently proposed in [5, 6]. In addition, we take advantage of the enriched framework to make methods local mass conservative and super convergent for some particular meshes. Numerical tests infer achieved theoretical results and validate optimal error estimates.

References

[1] Araya, R. and Barrenechea, G.R. and Valentin, F., *Stabilized finite element methods based on multiscale enrichment for the Stokes problem*, SIAM J. Numer. Anal., vol. 44, 1, pp. 322–348, (2006).

[2] Allendes, A., Barrenechea, G.R., Hernández, E., Valentin, F., *A two-level enriched finite element method for a mixed problem*, LNCC Research Report, (2008).

*This research was supported by CNPq, CAPES and FAPERJ/Brazil and NSF/USA

†Department of Mathematics, University of Strathclyde, 26 Richmond Street, Glasgow G1 1XH, UK, e-mail: grb@maths.strath.ac.uk

‡Department of Mathematical Sciences, University of Colorado at Denver, P.O. Box 173364, Campus Box 170 Denver, Colorado 80217-3364, USA, e-mail: leo.franca@cudenver.edu

§Department of Mathematical Sciences, University of Colorado at Denver, P.O. Box 173364, Campus Box 170 Denver, Colorado 80217-3364, USA, e-mail: charder@cudenver.edu

¶LNCC, Applied Mathematics Department, Av. Getulio Vargas 333, 25651-075 Petrópolis - RJ, Brazil, e-mail: valentin@lncc.br

[3] Barrenechea, G.R., Franca, L.P. and Valentin, F., A symmetric nodal conservative finite element method for the Darcy equation, LNCC Research Report, (2008).

[4] Barrenechea, G.R., Franca, L.P. and Valentin, F., *A Petrov-Galerkin enriched method for the Darcy equation*, Comput. Methods Appl. Mech. Engrg., vol. 25, 6, pp. 1237–1271, (2007).

[5] Bochev, P. and Dohrmann, C. R. and Gunzburger, M. D. *Stabilization of low-order mixed finite elements for the Stokes equations*, SIAM J. Numer. Anal., vol. 44, 1, pp. 82–101, (2006).

[6] Burman, E., *Pressure projection stabilizations for Galerkin approximations of Stoke' and Darcy's problem*, Numerical Methods for Partial Differential Equations, vol. 24, pp. 127–143, (2008).

A domain decomposition method for linear exterior boundary value problems in elasticity*

Mauricio A. Barrientos † Mario E. Mellado ‡

Abstract

In this paper we present new domain decomposition methods for solving linear exterior boundary value problems in elasticity in the plane. Our method is based on the combination of finite element method and Dirichlet–to–Neumann mapping, given in terms of Fourier series, which gives the exact boundary condition on an artificial boundary, to transform the exterior problem into an equivalent mixed boundary value problem in a bounded domain. As a model problem we consider the exterior boundary value problem for the Lamé system. The domain is decomposed into a finite number of subdomains and the Dirichlet data on the interfaces is introduced as the unknown of the associated discrete Steklov–Poincaré problem. Next, we use either a preconditioned Richardson–type method or the preconditioned conjugate gradient method by introducing adjustable Dirichlet–Robin–type preconditioners to solve the problem, which yields iteration–by–subdomains algorithms well suited for parallel computations and they can be naturally implemented on a parallel computing environment. A complete discrete analysis proves that our algorithms have an independent convergence of the stepsize of the mesh.

References

[1] M. A. Barrientos and M. E. Mellado, *Steklov-Poincaré operators and domain decomposition for solving linear exterior boundary value problems. Submitted.*

[2] H. Fujita, N. Saito, T. Suzuki, *Studies in Mathematics and its Applications, 30, Operator Theory and Numerical Methods.* Elsevier Science B.V., 2001.

*Work partially suported by Fondecyt Chile through the project No. 1070952

†Instituto de Matemáticas, Pontificia Universidad Católica de Valparaíso, Blanco Viel 596, Valparaíso, Chile, e-mail: mauricio.barrientos@ucv.cl

‡Centro de Investigación Científico Tecnológico para la Minería (CICITEM), Avenida Universidad de Antofagasta 02800, Antofagasta, Chile, e-mail: mmellado@uantof.cl

[3] G.N. Gatica and G.C. Hsiao, *The uncoupling of boundary integral and finite element methods for nonlinear boundary value problems.* Journal of Mathematical Analysis and Applications, vol. 189, 2, pp. 442–461, 1995.

[4] G. N. Gatica, E. C. Hernández and M. E. Mellado, *A domain decomposition method for linear exterior boundary value problems.* Applied Mathematics Letters, vol. 11, 6, pp. 1–9, (1998).

[5] H. Han and X. Wu, *The approximation of the exact boundary conditions at an artificial boundary for linear elastic equations and its applications.* Mathematics of Computation, vol. 59, pp. 21–37, (1992).

A stabilized mixed discontinuous Galerkin formulation: a priori and a posteriori error analyses*

Tomás P. Barrios† Rommel Bustinza‡

Abstract

In this talk we present an a priori and a posteriori error analysis of a stabilized mixed discontinuous Galerkin formulation for elliptic problems. Our approach requires the introduction of suitable Galerkin least-squares terms (arising from constitutive and equilibrium equations), which allow us to look for the flux unknown in the local Raviart-Thomas space. The unique solvability of the discrete scheme is established avoiding the introduction of lifting operators and allow us to conclude that the rate of convergence of the error, measured in an appropriate norm, is optimal respect to the $h-$version. Furthermore, we apply Helmholtz decomposition to obtain a reliable and efficient a posteriori error estimate for our approach. Finally, we present several numerical experiments, showing the robustness of the method as well as the theoretical properties of the estimator, thus confirming the capability of the corresponding adaptive algorithms to localize the inner layers, the singularities and/or the large stress regions of the exact solution.

References

[1] D.N. Arnold, F. Brezzi, B. Cockburn, L.D. Marini: *Unified analysis of dicontinuous Galerkin methods for elliptic problems.* SIAM Journal on Numerical Analysis, vol. 39, 5, pp. 1749-1779, (2001).

[2] T.P. Barrios and R. Bustinza: *An augmented discontinuous Galerkin method for elliptic problems.* Comptes Rendus de l'Academie des Sciences, Series I, vol. 344, pp. 53-58, (2007).

*This research was partially supported by FONDECYT Grants 1080168 and 11060014, by FONDAP and BASAL projects CMM, Universidad de Chile, and by Centro de Investigación en Ingeniería Matemática (CI^2MA), Universidad de Concepción.

†Departamento de Matemática y Física Aplicadas, Facultad de Ingeniería, Universidad Católica de la Santísima Concepción, Casilla 297, Concepción, Chile, e-mail: tomas@ucsc.cl

‡CI^2MA and Departamento de Ingeniería Matemática, Facultad de Ciencias Físicas y Matemáticas, Universidad de Concepción, Casilla 160-C, Concepción, Chile, e-mail: rbustinz@ing-mat.udec.cl

[3] T.P. Barrios and R. Bustinza: *A priori and a posteriori error analyses of an augmented Galerkin discontinuous formulation.* IMA Journal of Numerical Analysis, to appear.

[4] T.P. Barrios and R. Bustinza: *An augmented DG scheme for porous media equations.* Numerical Mathematics and Advanced Applications: Proceedings ENUMATH 2007, K. Kunisch, G. Of and O. Steinbach, eds, pp. 315-322, Springer Verlag, 2008.

[5] T.P. Barrios and R. Bustinza: *An a posteriori error analysis of an augmented discontinuous Galerkin formulation for Darcy flow.* Preprint 08-18, Departamento de Ingeniería Matemática, Universidad de Concepción, (2008).

[6] R. Becker, P. Hansbo and M.G. Larson: *Energy norm a posteriori error estimation for discontinuous Galerkin methods.* Computer Methods in Applied Mechanics and Engineering, vol. 192, pp. 723-733, (2003).

An *a posteriori* error analysis for the stream function and vorticity formulation of the Stokes Problem*

Tomás P. Barrios † José M. Cascón ‡ Galina C. García §

Abstract

In this paper we describe an a posteriori error estimator of the finite element solution for Stokes problem in stream function and vorticity formulation. We derive a reliable and efficient *a posteriori* error estimator. Our approach introduce an appropriate dual problem that allow us to prove efficiency in natural norms. In this sense, it can be seen as an extension of the applicability of the error indicator developed in [1] to the standard stream function-vorticity formulation. We present several numerical experiments confirming the theoretical properties of the estimator, and illustrating the capability of the corresponding adaptive algorithm to localize the singularities and the large stress regions of the solution. Finally, we apply the adaptive strategy to a large-scale ocean circulation model to reduce the spurious oscillations which arise when convective terms are dominant.

References

[1] M. Amara, M. Ben Younes and Bernardi, C., *Error indicators for the Navier-Stokes equations in stream function and vorticity formulation.* Numerische Mathematik, vol. 80, pp. 181-206, (1998).

[2] V. Girault and P.-A. Raviart, *Finite Element Methods for Navier Stokes Equations: Theory and Algorithms.* Springer-Verlang, Berlin, 1986.

*This research was partially supported by FONDECYT Grants No. 11060014 and 1080244 (Chile) and Grant CGL2008-06003-C03-03/CLI (Spain).

†Departamento de Matemática y Física Aplicada, Facultad de Ingeniería, Universidad Católica de la Santísima Concepción, Casilla 297, Concepción, Chile, e-mail: `tomas@ucsc.cl`

‡Departamento de Matemática, Universidad de Salamanca, 37008, Salamanca, España, e-mail: `casbar@usal.es`

§Departamento de Matemáticas y Ciencia de la Computación, Facultad de Ciencia, Universidad de Santiago, Casilla 307, Correo 2, Santiago, Chile, e-mail: `galina@fermat.usach.cl`

[3] A. SCHMIDT AND K. G. SIEBERT., *Design of Adaptive Finite Element Software: The Finite Element Toolboox ALBERTA*, LNCSE 42, Springer-Verlag Berlin Heidelberg 2005.

[4] R. VERFÜRTH, A Review of A Posteriori Error Estimation and Adaptive Mesh-Refinement Techniques. Wiley-Teubner (Chichester), 1996.

Convergence of a finite volume method for cardiac propagation*

Mostafa Bendahmane † Raimund Bürger ‡ Ricardo Ruiz §

Abstract

A finite volume method for solving the monodomain and bidomain models for the electrical activity of myocardial tissue is presented. These models consist of a parabolic PDE and a system of a degenerate parabolic and an elliptic PDE, respectively, for certain electric potentials, coupled to an ODE for the gating variable. Existence and uniqueness of the approximate solution is proved. It is also shown that the scheme converges to the corresponding weak solution for the monodomain model, and also for the bidomain equations in the special case of fibers aligned with the axis. Numerical examples in two and three space dimensions indicate experimental rates of convergence slightly above first order for both models. In addition, since typical solutions of the studied models exhibit wavefronts with steep gradients, the finite volume scheme is enriched by a fully adaptive multiresolution method, whose basic purpose is to concentrate computational effort on zones of strong variation of the solution. Time adaptivity is achieved by two alternative devices, namely locally varying time stepping and a Runge-Kutta-Fehlberg-type adaptive time integration. Finally, the optimal choice of the threshold for discarding non-significant information in the multiresolution representation of the solution is addressed.

References

[1] M. Bendahmane, R. Bürger and R. Ruiz, *Convergence of a finite volume scheme for cardiac propagation*. Submitted.

*This research was partially supported by Fondecyt projects 1050728, 1070682, Fondap in Applied Mathematics, project 15000001, BASAL project CMM, Universidad de Chile, Centro de Investigación en Ingeniería Matemática (CI^{2}MA), Universidad de Concepción, and by Conicyt Fellowship.

†Department of Mathematics, Al Imam University, Saudi Arabia, e-mail: mostafab@ing-mat.udec.cl

‡CI^{2}MA and Departamento de Ingeniería Matemática, Facultad de Ciencias Físicas y Matemáticas, Universidad de Concepción, Casilla 160-C, Concepción, Chile, e-mail: rburger@ing-mat.udec.cl

§Departamento de Ingeniería Matemática, Facultad de Ciencias Físicas y Matemáticas, Universidad de Concepción, Casilla 160-C, Concepción, Chile, e-mail: rruiz@ing-mat.udec.cl

[2] M. Bendahmane, R. Bürger and R. Ruiz, *A multiresolution space-time adaptive scheme for the bidomain model in electrocardiology.* Preprint 2008-10, Depto. Ing. Mat., U. de Concepción, available from `http://www.ing-mat.udec.cl/`; submitted.

[3] M. Bendahmane and K.H. Karlsen, *Analysis of a class of degenerate reaction-diffusion systems and the bidomain model of cardiac tissue.* Netw. Heterog. Media 1 (2006) 185–218.

[4] Y. Bourgault, Y. Coudière and C. Pierre, *Existence and uniqueness of the solution for the bidomain model used in cardiac electro-physiology.* Nonlin. Anal. Real World Appl. 10 (2009) 458–482.

[5] R. Eymard, Th. Gallouët and R. Herbin, *Finite Volume Methods.* In: P.G. Ciarlet, J.L. Lions (eds.), Handbook of Numerical Analysis, vol. VII, North-Holland, Amsterdam, 713–1020 (2000).

[6] S. Müller and Y. Stiriba, *Fully adaptive multiscale schemes for conservation laws employing locally varying time stepping.* J. Sci. Comput. 30 (2007) 493–531.

A new p-interpolation operator for Raviart-Thomas elements and its application to the convergence analysis of the high-order BEM for electro-magnetic scattering*

Alex Bespalov † Norbert Heuer ‡

Abstract

Interpolation operators (or projectors onto corresponding polynomial spaces) are frequently used in the analysis of discrete methods for time-harmonic Maxwell equations. In particular, a proper choice of these operators is critical for the proof of the discrete compactness property which, together with an appropriate approximability condition, implies the convergence of finite element methods (FEM) for Maxwell eigenvalue problems. When the problem of electro-magnetic scattering (modelled by Maxwell's equations in the exterior domain) is reformulated as a boundary integral equation on the surface of the scatterer, one can apply the boundary element method (BEM) for its approximate solution. The electric field integral equation (EFIE) is one of many possible integral formulations. It is usually discretised by the div-conforming Galerkin BEM based on Raviart-Thomas spaces. Then, as in the FEM for Maxwell's equations, an appropriate interpolation operator is needed to prove convergence of boundary element approximations for the EFIE. However, the existing operators do not easily fit the theoretical framework of the BEM, which is based on negative order Sobolev spaces. This is especially true for high-order methods (p- and hp-BEM).

In this talk we introduce a new $\tilde{\mathbf{H}}^{-1/2}(\mathrm{div})$-conforming p-interpolation operator for Raviart-Thomas elements. This operator has a number of useful properties related to the high-order BEM on piecewise smooth surfaces:

- it assumes sufficiently low $\mathbf{H}^r \cap \tilde{\mathbf{H}}^{-1/2}(\mathrm{div})$-regularity ($r > 0$);
- it commutes with the $\tilde{H}^{-1/2}$-projector;
- it is quasi-stable with respect to polynomial degrees.

Then we apply this interpolation operator to prove convergence of the hp-version of the BEM for the EFIE on piecewise plane (open or closed) surfaces discretised by quasi-uniform meshes.

*This research was supported by EPSRC (UK) under grant no. EP/E058094/1.

†Department of Mathematical Sciences, Brunel University, Uxbridge, West London UB8 3PH, UK, e-mail: `albespalov@yahoo.com`

‡Facultad de Matemáticas, Pontificia Universidad Católica de Chile, Avenida Vicuña Mackenna 4860, Santiago, Chile, e-mail: `nheuer@mat.puc.cl`

Numerical methods for an inverse problem in scalar conservation laws *

RAIMUND BÜRGER † ANIBAL CORONEL ‡ MAURICIO SEPÚLVEDA†

Abstract

This contribution is concerned with the numerical solution of flux identification problem in scalar conservation laws where the solution in a fixed time is known. This inverse problem is formulated in a variational setting by introducing an objective function which compares, in the L^2-norm, the simulation and the observation profiles. We consider two numerical methodologies to evaluate the exact gradient of the discrete objective function: the sensitivity equation method and the adjoint equation method. We comment the consequence of shock formation in the differentiability of the cost function in both continuous gradient formulations. In the case of the adjoint sensitivity analysis we interpret the continuous adjoint equation in the sense of reversible solutions and we prove the convergence of the exact gradient to an element of the subdifferential of the cost function. Although, the numerical examples are focus on the context of well-known phenomenological sedimentation model, the identification method can be applied to other one-dimensional hyperbolic models.

References

[1] BÜRGER A., CORONEL, A., AND SEPULVEDA, M. *A semi-implicit monotone difference scheme for an initial- boundary value problem of a strongly degenerate parabolic equation modelling sedimentation-consolidation processes.* Math. Comp. **75** (2006) 91–112.

[2] CORONEL, A., JAMES, F. AND SEPULVEDA, M. *Numerical identification of parameters for a model of sedimentation processes.* Inverse Problems **19** (2003) 951–972.

*This research was partially supported by Conicyt through Fondap in Applied Mathematics, Fondecyt 1070694, Fondecyt 1050728 and Fondecyt 11060400.

†Departamento de Ingeniería Matemática, Universidad de Concepción, Casilla 160-C, Concepción, Chile. E-mail: rburger@ing-mat.udec.cl, mauricio@ing-mat.udec.cl

‡Departamento de Ciencias Básicas, Facultad de Ciencias, Universidad del Bío-Bío, Casilla 447, Campus Fernando May, Chillán, Chile.
e-mail: acoronel@roble.fdo-may.ubiobio.cl

[3] James, F. and Sepúlveda, M. *Convergence results for the flux identification in a scalar sonservation law.* SIAM J. Control Optim. **37** (1999) 869–891.

[4] Glass, O., *An extension of Oleinik's inequality for general 1-D scalar conservation laws*, Journal of Hyperbolic Differential Equations, vol. 5, 1, pp. 113-165, (2008)

An entropy inequality for a class of multi-species kinematic flow models with discontinuous flux*

Raimund Bürger† Kenneth H. Karlsen‡ John D. Towers§

Abstract

We study a system of conservation laws that models multi-species kinematic flow models with an emphasis on models of multiclass traffic flow [1] and of the creaming of oil-in-water dispersions [6]. The flux is allowed to have a spatial discontinuity which models abrupt changes of road surface conditions or of the cross-sectional area in a settling vessel. For this system, an entropy inequality is proposed that singles out the correct solution at the interface. It is shown that limit solutions generated by a numerical scheme the authors recently proposed [2] satisfy this entropy inequality. It is also shown that limit solutions are entropy admissible, and in the genuinely nonlinear case, satisfy the usual Lax condition for a shock located away from the interface. We present an improvement to our scheme, involving a special interface flux that is activated only at a few grid points where the flux discontinuity is located. Numerical experiments indicate that this interface flux essentially eliminates overshoots that are sometimes present at the interface with our original scheme. We show that the scheme, with or without the interface fix, preserves a natural invariant region. Related earlier work includes [3, 5, 7]. This contribution is based on work under preparation [4].

References

[1] S. Benzoni-Gavage and R.M. Colombo, An n-populations model for traffic flow. *Eur. J. Appl. Math.*, 14:587–612, 2003.

*This research was partially supported by Fondecyt 1050728 FONDAP and BASAL projects CMM, Universidad de Chile, and by Centro de Investigación en Ingeniería Matemática (CI^{2}MA), Universidad de Concepción.

†CI^{2}MA and Departamento de Ingeniería Matemática, Facultad de Ciencias Físicas y Matemáticas, Universidad de Concepción, Casilla 160-C, Concepción, Chile, e-mail: rburger@ing-mat.udec.cl

‡Centre of Mathematics for Applications (CMA), University of Oslo, P.O. Box 1053, Blindern, N–0316 Oslo, Norway, e-mail: kennethk@math.uio.no

§MiraCosta College, 3333 Manchester Avenue, Cardiff-by-the-Sea, CA 92007-1516, USA, e-mail: john.towers@cox.net

[2] R. Bürger, A. García, K.H. Karlsen, and J. D. Towers, A family of schemes for kinematic flows with discontinuous flux. *J. Engrg. Math.*, 60:387–425, 2008.

[3] R. Bürger, A. García, K. H. Karlsen, and J. D. Towers, Difference schemes, entropy solutions, and speedup impulse for an inhomogeneous kinematic traffic flow model. *Netw. Heterog. Media*, 3:1–41, 2008.

[4] R. Bürger, K.H. Karlsen, and J. D. Towers, An entropy inequality for a class of multi-species kinematic flow models with discontinuous flux. In preparation.

[5] R. Donat and P. Mulet, Characteristic-based schemes for multi class Lighthill-Witham-Richards traffic models. *J. Sci. Comput.* 37:233–250, 2008.

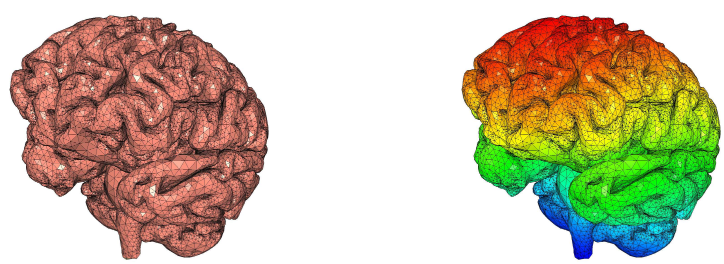
[6] F. Rosso and G. Sona, Gravity-driven separation of oil-water dispersions. *Adv. Math. Sci. Appl.*, 11:127–151, 2001.

[7] P. Zhang, S.C. Wong, and Z. Xu, A hybrid scheme for solving a multi-class traffic flow model with complex wave breaking. *Comput. Meth. Appl. Mech. Engrg.*, 197:3816–3827, 2008.

Modelling and simulation of the viscoelastic behavior of brain structures: preliminary results*

Maya de Buhan † Pascal Frey ‡

Abstract
We consider the problem of modelling the deformation of brain structures for which the nonlinear viscoelastic behavior has been established several years ago by [2]. Based on the thorough mathematical analysis of a model with internal variable suggested by [1], we focus here on its implementation in three dimensions. The problem associates a nonlinear PDE endowed with an incompressibility condition and an ODE describing the time evolution of the internal variable. The time discretization is based on an implicit Euler scheme and the spatial discretization involves $\mathbb{P}_2$ Lagrange finite elements. A linearized version of the resulting system is obtained by a Newton method and is solved by an Augmented Lagrangian technique. Computational results on complex domains will be provided to emphasize the adaptation on the geometric properties of the domain boundaries. Provided biophysical coefficients are available, these results aim to be confronted to experimental results in order to validate the underlying model. Furthermore, a coupling of this model with a plasticity model can be envisaged, possibly in other applications.



References

[1] Le Tallec P., *Numerical Analysis of viscoelastic problems*, Coll. Recherches en Mathmatiques Appliques, vol. 15, Masson, Paris (1990).

*This research was partially supported by CNRS, CONICYT and EGIDE.

†Universidad de Chile, UMI 2807, Departamento de Ingeniería Matemática, Santiago, Chile, e-mail: debuhan@dim.uchile.cl

‡UPMC Universit Paris-06, UMR 7598, Laboratoire Jacques-Louis Lions, 75005 Paris, France, e-mail: frey@ann.jussieu.fr

[2] Wittek A. et al., *Patient-specific model of brain deformation : Application to medical image registration*, Journal of Biomechanics, vol. 40, 4, p. 919-929, (2006).

Numerical simulation of two-dimensional Bingham fluid flow by semismooth Newton methods*

JUAN CARLOS DE LOS REYES[†] SERGIO GONZÁLEZ[‡]

Abstract

This work is devoted to the numerical simulation of two-dimensional stationary Bingham fluid flow by semi-smooth Newton methods. Bingham fluids are visco-plastic materials which behave like incompressible fluids in the regions where the stress reaches a given yield and like solids in the regions where the stress remains below that threshold. The mathematical models for such materials involve the constituent law for viscous incompressible fluids, with an extra stress tensor component modeling the visco-plastic effects. We are concerned with Bingham fluid flow in a given domain $\Omega \subset \mathbb{R}^2$, considering non-homogeneous Dirichlet and stress-free boundary conditions. We analyze the modeling elliptic variational inequality of the second kind as an equivalent minimization problem and, using Fenchel's duality, we obtain an optimality system which characterizes the primal and dual solutions. Since the solution to the dual problem is not unique, a family of Tikhonov regularized problems is introduced and the convergence of the regularized solutions to the original one is studied. For the discretization of each regularized optimality system, a finite element method with (cross-grid $\mathbb{P}_1$)$-\mathbb{Q}_0$ elements is utilized. The chosen pair is known to satisfy the Ladyzhenskaya - Babuška - Brezzi condition and allows also to obtain a direct relation between the discrete primal and dual variables. For the solution of the resulting system of nonsmooth equations, we propose a semismooth Newton algorithm. Using an additional relaxation of the incompressibility condition a modified reduced system matrix is constructed and a descent direction is obtained from each semismooth Newton iteration. The local superlinear convergence of the method is also proved. Finally, several numerical experiments are carried out in order to investigate the behavior and efficiency of the method.

*Research partially supported by DAAD, EPN-Quito and TU-Berlin joint project: "PhD Programm in Applied Mathematics".

[†]Departamento de Matemática, EPN Quito, Ladrón de Guevara E11-253, Quito, Ecuador, e-mail: jcdelosreyes@epn.edu.ec

[‡]Departamento de Matemática, EPN Quito, Ladrón de Guevara E11-253, Quito, Ecuador, e-mail: sgonzalez@epn.edu.ec

References

[1] De los Reyes, J.C., and González, S., *Path following methods for steady laminar Bingham flow in cylindrical pipes.* ESAIM: Mathematical Modelling and Numerical Analysis, to appear.

[2] Glowinski, R., *Numerical methods for Nonlinear Variational Problems.* Springer-Verlar, (1982).

[3] Hintermüller, M., and Ito, K., and Kunisch, K., *The primal-dual active set strategy as a semismooth Newton method.* SIAM J. Optim., 13, pp. 868-888, (2002).

[4] Hintermüller, M., and Stadler, G., *An infeasible primal-dual algorithm for TV-based inf-convolution-type image restoration.* SIAM J. Sci. Comp., 28, pp. 1-23, (2006).

[5] Mosolov, P.P. and Miasnikov, V.P., *Variational Methods in the Theory of the Fluidity of a Viscous-Plastic Medium.* Journal of Applied Mathematics and Mechanics (P.M.M.), vol. 29, pp. 468-492, (1965).

Finite element error analysis for state-constrained optimal control of the Stokes equations

Juan C. de los Reyes [*] Christian Meyer [†] Boris Vexler [‡]

Abstract

An optimal control problem for 2d and 3d Stokes equations is investigated with pointwise inequality constraints on the state and the control. The paper is concerned with the full discretization of the control problem allowing for different types of discretization of both the control and the state. For instance, piecewise linear and continuous approximations of the control are included in the present theory. Under certain assumptions on the L^∞-error of the finite element discretization of the state, error estimates for the control are derived which can be seen to be optimal since their order of convergence coincides with the one of the interpolation error. The assumptions of the L^∞-finite-element-error can be verified for different numerical settings. Finally the results of two numerical experiments are presented.

References

[1] D.N. Arnold, X. Liu, *Local error estimates for finite element discretizations of the Stokes equations*, M2AN, 29 (1995), pp. 367–389.

[2] P. Bochev and M. Gunzburger, *Least-squares finite-element methods for optimization and control for the Stokes equations*, Comput. Math. Appl., 48 (2004), pp. 1035–1057.

[3] C. Carstensen, *Quasi-interpolation and a posteriori error analysis in finite elements methods*, M2AN, 33 (1999), pp. 1187–1202.

[*]Departamento de Matemática, Escuela Politécnica Nacional de Quito, Ladrón de Guevara E11-253, Quito, Ecuador, e-mail: juan.delosreyes@epn.edu.ec

[†]Weierstrass Institute for Applied Analysis and Stochastics (WIAS), Mohrenstr. 39, D-10117 Berlin, Germany, e-mail: meyer@wias-berlin.de

[‡]Technische Universität München Lehrstuhl für Mathematische Optimierung, Zentrum Mathematik, Boltzmannstr. 3, 85748 Garching b. München, Germany, e-mail: vexler@ma.tum.de

[4] M. Dauge, *Stationary Stokes and Navier-Stokes systems on two- and three-dimensional domains with corners. Part I: Linearized equations*, SIAM J. Math. Anal., 20 (1989), pp. 74–97.

[5] J.-C. de los Reyes and K. Kunisch, *A Semi-Smooth Newton Method for Control Constrained Boundary Optimal Control of the Navier-Stokes Equations*, Nonlinear Analysis: Theory, Methods & Applications, 62 (2005), pp. 1289–1316.

[6] R.G. Duran and R.H. Nochetto, *Weighted inf-sup condition and pointwise error estimates for the Stokes problem*, Math. Comp., 54 (1990), pp. 63–79.

[7] C. Meyer, *Error estimates for the finite-element approximation of an elliptic control problem with pointwise state and control constraints*, submitted.

[8] A. Rösch and B. Vexler, *Optimal Control of the Stokes Equations: A Priori Error Analysis for Finite Element Discretization with Postprocessing*, SIAM J. Numer. Anal., 44 (2006), pp. 1903–1920.

hp-adaptive finite elements for multiphysics wave propagation problems

Leszek F. Demkowicz *

Abstract

I will attempt to combine an overview of our experience with *hp* finite elements and the automatic *hp*-adaptivity for wave propagation problems, with a presentation of new research topics focusing on multiphysics coupled problems. The first part of the talk will focus on fundamentals of *hp*-discretizations of wave propagation problems: acoustics, elasticity and electromagnetics. We will shortly discuss the issues of stability and approximability for time-harmonic problems emphasizing the difference between the elliptic and Maxwell problems. I will review the main results of the theory of projection-based interpolation and discuss its importance in both the theoretical (proof of discrete compactness for *hp* methods) and practical (automatic *hp*-adaptivity) context. This part of the presentation will deliver "punch lines" only, and I will finish it by "flashing" a few representative examples. The second part of the presentation will address our current work on multiphysics coupled-problems. Using a coupled acoustics/elasticity problem, I will outline new challenges that we have faced when generalizing the *hp* methodology to this class of problems. This will include a discussion on *hp* data structures, use of fractional Sobolev norms, and both energy- and goal-driven automatic *hp*-adaptivity algorithms. The discussion will be illustrated with numerical solutions of 3D axisymmetric problems.

*Institute for Computational Engineering and Sciences, University of Texas at Austin, USA, e-mail: leszek@ices.utexas.edu

On the calculation of Maxwell's eigenfrequencies using integral equations for a buried landmine*

MARIO DURÁN † MILKO MATURANA ‡
JEAN-CLAUDE NÉDÉLEC § SEBASTIÁN OSSANDÓN ¶

Abstract
A 3-D mathematical model, using the integral equation framework, of the time-harmonic Maxwell's equations has been developed for research studies of buried penetrable targets in a dispersive isotropic soil. An efficient numerical method is developed to calculate precisely the Maxwell's eigenfrequencies of buried landmines, located in a given high frequency interval. Functions are evaluated only in the boundary of the domain, so very fine discretizations may be chosen to obtain high eigenfrequencies. We discuss the stability and convergence of the proposed method. Finally we show some numerical results, which make evident the effectiveness and relevance of the proposed numerical method.

References

[1] BENDALI A, DEVYS C., *Calcul numérique du rayonnement de cornets électromagnétiques dont l'ouverture est partiellement remplie par un diélectrique.* L'Onde électrique 1986; **66**:77-81.

[2] MIKHLIN S.G., Integral Equations. Pergamon Press: Oxford, 1957.

[3] NÉDÉLEC J.-C., Acoustic and Electromagnetic Equations: Integral Representations for Harmonic Problems. Springer-Verlag, 2001.

*Work partially supported by Proyecto ANILLO de Investigación ACT30 del Programa Bicentenario en Ciencia y Tecnología de CONICYT/World Bank (Chile).

†Facultad de Ingeniería, Pontificia Universidad Católica de Chile, Casilla 306, Santiago 22, Chile, e-mail: mduran@ing.puc.cl

‡Academia Politécnica Militar, Valenzuela Llanos 623, La Reina, Santiago, Chile, e-mail: mematura@uc.cl

§Centre de Mathématiques Appliquées, Ecole Polytechnique, 91128 Palaiseau Cedex, France, e-mail: nedelec@cmap.polytechnique.fr

¶Instituto de Matemática, Pontificia Universidad Católica de Valparaíso, Casilla 4059, Valparaíso, Chile, Chile, e-mail: sebastian.ossandon@ucv.cl

A radiation condition for time-harmonic elastic waves in half-spaces with free boundary*

MARIO DURÁN † IGNACIO MUGA ‡ JEAN-CLAUDE NÉDÉLEC §

Abstract

In this work we deduce an explicit Sommerfeld-type radiation condition able to prove uniqueness for the problem of outgoing wave propagation in isotropic elastic half-spaces with free boundary condition. The expression is obtained by a rigorous asymptotic analysis of the Green's function associated with this problem. Observe that this is an exterior problem with unbounded frontier. The main difficulty is that the free boundary condition allows the propagation of Rayleigh waves guided by the unbounded surface. Hence, we mainly observe three types of waves in the far field expansion (each one with its own velocity) : **the pressure wave, the shearing wave and the Rayleigh or surface wave**. Thus, the outgoing wave behavior needs to be described by a radiation condition different from the usual Kupradze's condition [2], which is used in exterior problems with bounded boundaries (where we only see pressure and shearing waves in the far field). This is an extension to the elastic case of the previous result in [1].

References

[1] DURÁN, M., MUGA, I. AND NÉDÉLEC, J.-C., *The Helmholtz Equation in a Locally Perturbed Half-Space with Non-Absorbing Boundary.* Archive for Rational Mechanics and Analysis, Vol. 191, 1, pp 143-172 , (2009).

[2] KUPRADZE, V.D., *Three-dimensional Problems of the Mathematical Theory of Elasticity and Thermoelasticity.* North-Holland Series in Applied Mathematics and Mechanics, vol. 25, New York, 1979.

*This research is supported by the project *Anillo ACT30*, CONICYT/World Bank (Chile).

†Centro de Minería, Pontificia Universidad Católica de Chile, e-mail: maduran@ing.puc.cl

‡Instituto de Matemáticas, Pontificia Universidad Católica de Valparaíso, Chile, e-mail: ignacio.muga@ucv.cl

§Centre de Mathématiques Appliquées, École Polytechnique, France, e-mail: nedelec@cmapx.polytechnique.fr

Existence, uniqueness and approximation of the Stokes equations in some non-Lipschitz domains*

Ricardo G. Durán† Fernando López-García‡

Abstract

Theoretical and numerical analysis of elliptic problems are strongly based on several results in the theory of Sobolev spaces which are valid for Lipschitz domains (trace theorems, Korn inequality, inf-sup conditions, etc.) On the other hand, it is known that for some non-Lipschitz domains many of these results are not valid. In this talk we consider a particular class of non-Lipschitz domains, namely, domains with external cusps (for example the complement of two tangent circles or spheres). This kind of domains can be viewed as a particular case of Hölder-α domains. First we recall our previous works [1, 2] on the analysis and approximation of the Poisson equation in this kind of domains. Second, we present new results for the Stokes equations. As it is well known, existence, uniqueness and stability of numerical solutions follows from the inf-sup condition and its discrete counterpart. We show by an elementary example that the standard inf-sup condition does not hold for the domains that we are considering. Therefore, a natural question is whether some weaker inf-sup condition can be proved for these domains. To give a positive answer to this question we work with weighted Sobolev norms. We present a result for the general class of Hölder-α domains [3] and a sharper result for particular domains with external cusps. These generalized inf-sup conditions allow us to apply the general theory of saddle point problems to the analysis of the Stokes equations in these domains.

References

[1] Acosta, G., Armentano, M. A., Durán, R. G and Lombardi, A. L., *Nonhomogeneous Neumann problem for the Poisson equation in domains with an external cusp*, Journal of Mathematical Analysis and Applications, vol. 310, 2, pp. 397-411, (2005).

*This research was partially supported by CONICET and University of Buenos Aires, Argentina.

†Departamento de Matemática, Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires, Argentina, e-mail: rduran@dm.uba.ar

‡Departamento de Matemática, Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires, Argentina, e-mail: flopezg@dm.uba.ar

[2] Acosta, G., Armentano, M. A., Durán, R. G and Lombardi, A. L., *Finite element approximations in a non-Lipschitz domain*, SIAM J. Numer. Anal., vol. 45, 1, pp. 277-295, (2007).

[3] Durán, R. G and López-García, F., *A right inverse of the divergence for planar Hölder-α*, submitted to Math. Mod. Meth. Appl. Sci..

Computation of the vibration modes of a Reissner-Mindlin laminated plate

Ricardo G. Durán* Rodolfo Rodríguez† Frank Sanhueza‡

Abstract

This work deals with the computation of the vibration modes of a laminated plate modeled by the Reissner-Mindlin equations [1], by using the DL3 elements for the bending terms and linear triangular finite elements for the in-plane displacements [2, 4]. We apply the general approximation theory for spectral problems and, under appropriate assumptions, we obtain optimal order error estimates, including a double order for the vibration frequencies. The estimates are independent of the thickness of the laminated plate, which leads to the conclusion that the method is locking-free [3]. Numerical tests are reported to assess the performance of the method.

References

[1] F. Auricchio, C. Lovadina and E. Sacco, *Analysis of mixed finite elements for laminated composite plates*, in *Comput. Methods Appl. Mech. Engrg.* **190** 4767–4783.

[2] R.Durán and E. Liberman, *On mixed finite element methods for Reissner Mindlin Plate model*, in Math. Comp., **58** 561-573.

[3] D.N. Arnold and R.S. Falk, *A uniformly accurate finite element method for the Reissner-Mindlin plate*, SIAM J. Numer. Anal., **26** (1989) 1276–1290.

[4] R. Durán, L. Hervella-Nieto, E. Liberman, R. Rodríguez and J. Solomin, *Approximation of the vibration modes of a plate by Reissner-Mindlin equations*, Math. Comp., **68** (1999) 1447–1463.

*Departamento de Matemática, Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires, 1428 - Buenos Aires, Argentina. Partially supported by CONICET under grant PIP 5478 and Universidad de Buenos Aires under grant X052. Member of CONICET (Argentina).

†CI²MA, Departamento de Ingeniería Matemática, Universidad de Concepción, Casilla 160-C, Concepción, Chile. Partially supported FONDAP and BASAL projects CMM, Universidad de Chile.

‡Departamento de Ingeniería Matemática, Universidad de Concepción, Casilla 160-C, Concepción, Chile. Partially supported by CONICYT (Chile)

A residual-based a posteriori error estimator for a two-dimensional fluid-solid interaction problem*

GABRIEL N. GATICA † GEORGE C. HSIAO ‡ SALIM MEDDAHI §

Abstract

In this paper we develop an a posteriori error analysis of a mixed finite element method for a fluid-solid interaction problem posed in the plane. The media are governed by the acoustic and elastodynamic equations in time-harmonic regime, respectively, and the transmission conditions are given by the equilibrium of forces and the equality of the normal displacements of the solid and the fluid. The coupling of primal and dual-mixed finite element methods is applied to compute both the pressure of the scattered wave in the linearized fluid and the elastic vibrations that take place in the elastic body. The finite element subspaces consider continuous piecewise linear elements for the pressure and a Lagrange multiplier defined on the interface, and PEERS for the stress and rotation in the solid domain. We derive a reliable and efficient residual-based a posteriori error estimator for this coupled problem. Suitable auxiliary problems, the continuous inf-sup conditions satisfied by the bilinear forms involved, a discrete Helmholtz decomposition, and the local approximation properties of the Clément interpolant and Raviart-Thomas operator are the main tools for proving the reliability of the estimator. Then, Helmholtz decomposition, inverse inequalities, and the localization technique based on triangle-bubble and edge-bubble functions are employed to show the efficiency.

References

[1] BERNARDI, C., CANUTO, C. AND MADAY, Y., *Generalized inf-sup conditions for Chebyshev spectral approximations of the Stokes problem.* SIAM Journal on Numerical Analysis, vol. 25, 6, pp. 1237-1271, (1988).

*This research was partially supported by FONDAP and BASAL projects CMM, Universidad de Chile, and by Centro de Investigación en Ingeniería Matemática (CI^{2}MA), Universidad de Concepción.

†CI^{2}MA and Departamento de Ingeniería Matemática, Facultad de Ciencias Físicas y Matemáticas, Universidad de Concepción, Casilla 160-C, Concepción, Chile, e-mail: ggatica@ing-mat.udec.cl

‡Department of Mathematical Sciences, College of Arts and Sciences, University of Delaware, Newark, DE, USA, e-mail: hsiao@math.udel.edu

§Departamento de Matemáticas, Facultad de Ciencias, Universidad de Oviedo, Calvo Sotelo s/n, Oviedo, España, e-mail: salim@uniovi.es

[2] Gatica, G.N. and Maischak, M., *A posteriori error estimates for the mixed finite element method with Lagrange multipliers*. Numerical Methods for Partial Differential Equations, vol. 21, 3, pp. 421-450, (2005).

[3] Gatica, G.N., Márquez, A. and Meddahi, S., *Analysis of the coupling of primal and dual-mixed finite element methods for a two-dimensional fluid-solid interaction problem*. SIAM Journal on Numerical Analysis, vol. 45, 5, pp. 2072-2097, (2007).

[4] Hsiao, G.C., Kleinman, R.E. and Schuetz, L.S., *On variational formulations of boundary value problems for fluid-solid interactions*. In: Elastic Wave Propagation, M.F. McCarthy and M.A. Hayes (Editors), pp. 321-326, Elsevier Science Publishers B.V. (North-Holland), 1989.

[5] Meddahi, S. and Sayas F.-J., *Analysis of a new BEM-FEM coupling for two dimensional fluid-solid interaction*. Numerical Methods for Partial Differential Equations, vol. 21, 6, pp. 1017-1042, (2005).

[6] Verfürth, R., A Review of A Posteriori Error Estimation and Adaptive Mesh-Refinement Techniques. Wiley-Teubner (Chichester), 1996.

A coupled mixed finite element method for the interaction problem between electromagnetic field and elastic body*

Gabriel N. Gatica † George C. Hsiao ‡ Salim Meddahi §

Abstract

This paper deals with the coupled problem arising from the interaction of a time harmonic electromagnetic field with a three-dimensional elastic body. More precisely, we consider a suitable transmision problem holding between the solid and a sufficiently large annular region surrounding it, and aim to compute both the magnetic component of the scattered wave and the stresses that take place in the obstacle. To this end, we assume Voigt's model, which allows interaction only through the boundary of the body, and employ a dual-mixed variational formulation in the solid media. As a consequence, one of the two transmission conditions becomes essential, whence it is enforced weakly through the introduction of a Lagrange multiplier. The abstract framework developed in a recent work by A. Buffa is applied next to show that our coupled variational formulation is well posed. In addition, we define the corresponding Galerkin scheme by using PEERS in the solid and the edge finite elements of Nédélec in the electromagnetic region. Then, we prove that the resulting coupled mixed finite element scheme is uniquely solvable and convergent. Moreover, optimal a priori error estimates are derived in the usual way. Finally, some numerical results illustrating the analysis and the good performance of the method are also reported.

References

[1] Arnold, D.N., Brezzi, F. and Douglas, J., PEERS: *A new mixed finite element method for plane elasticity.* Japan Journal of Applied Mathematics, **1** (1984), 347-367.

---

*This research was partially supported by FONDAP and BASAL projects CMM, Universidad de Chile, and by Centro de Investigación en Ingeniería Matemática (CI^{2}MA), Universidad de Concepción.

†CI^{2}MA and Departamento de Ingeniería Matemática, Facultad de Ciencias Físicas y Matemáticas, Universidad de Concepción, Casilla 160-C, Concepción, Chile, e-mail: ggatica@ing-mat.udec.cl

‡Department of Mathematical Sciences, College of Arts and Sciences, University of Delaware, Newark, DE, USA, e-mail: hsiao@math.udel.edu

§Departamento de Matemáticas, Facultad de Ciencias, Universidad de Oviedo, Calvo Sotelo s/n, Oviedo, España, e-mail: salim@uniovi.es

[2] Buffa, A., *Remarks on the discretization of some non-coercive operator with applications to heterogeneous Maxwell equations.* SIAM Journal on Numerical Analysis, **43**(1) (2005), 1-18.

[3] Buffa, A. and Ciarlet, P., Jr., *On traces for functional spaces related to Maxwell's equation. Part* I: *An integration by parts formula in Lipschitz polyhedra.* Mathematical Methods in Applied Sciences **24**(1) (2001) 9-30.

[4] Buffa, A. and Ciarlet, P., Jr., *On traces for functional spaces related to Maxwell's equation. Part* II: *Hodge decompositions on the boundary of Lipschitz polyhedra and applications.* Mathematical Methods in Applied Sciences **21** (2001), 31-48.

[5] Buffa, A., Costabel, M. and Sheen, D., *On traces for* $\mathbf{H}(\mathbf{curl}, \Omega)$ *in Lipschitz domains.* Journal of Mathematical Analysis and Applications **276**(2) (2002), 845-867.

[6] Buffa, A. and Hiptmair, R. *Galerkin boundary element methods for electromagnetic scattering.* Topics in Computational Wave Propagation, 83-124, Lectures Notes in Computer Sciences and Engineering, 31, Springer, Berlin, 2003.

[7] Cakoni, F. and Hsiao, G.C., *Mathematical model of the interaction problem between electromagnetic field and elastic body.* Acoustics, Mechanics, and the Related Topics of Mathematical Analysis, 48-54, World Sci. Publ., River Edge, NJ, 2002.

[8] Gatica, G.N., Márquez, A. and Meddahi, S., *Analysis of the coupling of primal and dual-mixed finite element methods for a two-dimensional fluid-solid interaction problem.* SIAM Journal on Numerical Analysis **45**(5) (2007), 2072-2097.

[9] Hiptmair, R., *Finite elements in computational electromagnetism.* Acta Numerica 11 (2002), 237-339.

[10] Hiptmair, R., *Coupling of finite elements and boundary elements in electromagnetic scattering.* SIAM Journal on Numerical Analysis **14**(3) (2003), 237-339.

[11] Monk, P., Finite Element Methods for Maxwell's Equations. Oxford University Press, New York, 2003.

A residual-based a posteriori error estimator for a fully mixed formulation of the Stokes-Darcy coupled problem*

GABRIEL N. GATICA † RICARDO E. OYARZÚA ‡ FRANCISCO J. SAYAS§

Abstract

In this paper we develop an a posteriori error analysis of a fully mixed finite element method for the coupling of fluid flow with porous media flow. The flows are governed by the Stokes and Darcy equations, respectively, and the transmission conditions are given by mass conservation, balance of normal forces, and the Beavers-Joseph-Saffman law. The finite element subspaces consider Raviart-Thomas elements for the stress tensor of the Stokes equations, piecewise constants and Raviart-Thomas elements for the velocities, piecewise constants for the pressure in the porous medium, and continuous piecewise linear elements for the Lagrange multipliers defined on the interface. We derive a reliable and efficient residual-based a posteriori error estimator for this coupled problem. The proof of reliability makes use of Helmholtz decompositions and local approximation properties of the Clément interpolant and Raviart-Thomas operator. On the other hand, the localization technique based on triangle-bubble and edge-bubble functions constitute the main tools for proving the efficiency of the estimator. Finally, some numerical results illustrating the analysis and confirming the good performance of the corresponding adaptive algorithm are reported.

References

[1] I. BABUŠKA AND G.N. GATICA, *On the mixed finite element method with Lagrange multipliers*. Numerical Methods for Partial Differential Equations, vol. 19, 2, pp. 192-210, (2003).

*This research was partially supported by FONDAP and BASAL projects CMM, Universidad de Chile, and by Centro de Investigación en Ingeniería Matemática (CI^{2}MA), Universidad de Concepción.

†CI^{2}MA and Departamento de Ingeniería Matemática, Facultad de Ciencias Físicas y Matemáticas, Universidad de Concepción, Casilla 160-C, Concepción, Chile, e-mail: `ggatica@ing-mat.udec.cl`

‡Departamento de Ingeniería Matemática, Facultad de Ciencias Físicas y Matemáticas, Universidad de Concepción, Casilla 160-C, Concepción, Chile, e-mail: `royarzua@ing-mat.udec.cl`

§Departamento de Matemática Aplicada, Universidad de Zaragoza, Centro Politécnico Superior, María de Luna, 3 - 50018 Zaragoza, Spain, email: `jsayas@unizar.es`

[2] I. Babuška and G.N. Gatica, *A residual-based a posteriori error estimator for the Stokes-Darcy coupled problem.* Preprint 08-13, Departamento de Ingeniería Matemática, Universidad de Concepción, (2008).

[3] C. Carstensen, *A posteriori error estimate for the mixed finite element method.* Mathematics of Computation, vol. 66, 218, pp. 465-476, (1997).

[4] C. Carstensen, *An a posteriori error estimate for a first kind integral equation.* Mathematics of Computation, vol. 66, 217, pp. 139-155, (1997).

[5] C. Carstensen and G. Dolzmann, *A posteriori error estimates for mixed FEM in elasticity.* Numerische Mathematique, vol. 81, pp. 187-209, (1998).

[6] P. Clément, *Approximation by finite element functions using local regularisation.* RAIRO Modélisation Mathématique et Analyse Numérique, vol. 9, pp. 77-84, (1975).

[7] G.N. Gatica, *A note on the efficiency of residual-based a-posteriori error estimators for some mixed finite element methods.* Electronic Transactions on Numerical Analysos, vol 17, pp. 218-233, (2004).

[8] G.N. Gatica, S. Meddahi, and R. Oyarzúa, *A conforming mixed finite-element method for the coupling of fluid flow with porous media flow.* IMA Journal of Numerical Analysis 2008; doi: 10.1093/imanum/drm049.

[9] G.N. Gatica, R. Oyarzúa and F-J Sayas, *Convergence of a family of Galerkin discretizations for the Stokes-Darcy coupled problem.* Preprint 08-11, Departamento de Ingeniería Matemática, Universidad de Concepción, (2008).

[10] R. Verfürth, *A review of A posteriori error estimation and adaptive mesh-refinement thecniques* Wiley-Teubner (Chichester), 1996.

About computing the nonlinear interaction between weakly converging sequences and its influence in optimal design and nonlinear elasticity

Sergio Gutiérrez* Joaquín Mura†

Abstract

When solving a calculus of variations problem, very often one encounters weakly converging sequences that interact in a nonlinear manner. Two paradigmatic examples of this are the broad area of optimal design, see [1], and the characterization of the energy density functions to be used in nonlinear elasticity. For the latter we present the use of Compensated Compactness, [6], to derive a computational method to look for the elusive counterexample of a rank-one, non quasiconvex energy density in the planar case, see [4] and [5]. In the context of optimal design we present the small amplitude homogenization approach introduced in [2] and used later in [3], which is based on the use of the technique of H-measures introduced by L. Tartar in [7], to compute the interaction between the design variable and the state function.

References

[1] G. Allaire, *Shape Optimization by the Homogenization Method.* Springer-Verlag (2002).

[2] G. Allaire, S. Gutiérrez, *Optimal design in small amplitude homogenization.* ESAIM Math. Modelling and Numerical Analysis, 41(3) 543-574 (2007). Preprint available at http://www.cmap.polytechnique.fr/preprint/repository/576.pdf.

[3] S. Gutiérrez, J. Mura, *Small amplitude homogenization applied to inverse problems.* Computational Mechanics, 41(5) 699-706 (2008).

*Departamento de Ingeniería Estructural y Geotécnica, Pontificia Universidad Católica de Chile, Santiago Chile, e-mail: `sgutierr@ing.puc.cl`.

†Departamento de Ingeniería Estructural y Geotécnica, Pontificia Universidad Católica de Chile, Santiago Chile, e-mail: `jamura@ing.puc.cl`. Partially supported by Conicyt.

[4] S. Gutiérrez, J. Villavicencio, *An optimization algorithm applied to the Morrey conjecture in nonlinear elasticity.* Int. J. of Solids and Structures, 44(10) 3177-3186 (2007).

[5] S. Gutiérrez, *A Necessary Condition for the Quasiconvexity of Polynomials of Degree Four*. J. of Convex Analysis 13(1) 51-60 (2006).

[6] L. Tartar, *Compensated Compactness and Applications to Partial Differential Equations*. Nonlineal Analysis and Mechanics; Heriot-Watt Symposium Vol. IV (Knops R.J. ed.) Research Notes in Mathematics, No. 39, Pitman (1979).

[7] L. Tartar, *H-measures, a new approach for studying homogenization, oscillations and concentration effects in partial differential equations*, Proc. of the Royal Soc. Edinburgh, 115A 193-230 (1990).

Mortar boundary elements*

Martin Healey† Norbert Heuer‡

Abstract

Recently we started investigating non-conforming boundary element methods. The first paper [1] deals with the incorporation by Lagrangian multipliers of essential boundary conditions at the border of open surfaces in trace spaces of H^1. The second paper [2] analyzes the use of Crouzeix-Raviart elements for the discretization of hypersingular operators. Although in both cases there are no well-posed continuous formulations we proved that the discrete schemes converge almost quasi-optimally, that is, standard *a priori* error estimates are perturbed only by logarithmic terms. In this talk we deal with the more general case of domain decomposition in trace spaces of H^1 where continuity of approximating functions across interfaces is incorporated in a weak discrete sense. This strategy gives huge flexibility for discretizations which can be almost independent in individual sub-domains. Such a discretization is well-known for finite elements and is called mortar method. We consider this domain decomposition method for the discretization of hypersingular integral equations, prove its almost quasi-optimal convergence and present numerical results to underline the theory.

References

[1] Gatica, G.N., Healey, M. and Heuer, N., *The boundary element method with Lagrangian multipliers.* Accepted for publication in Numerical Methods for Partial Differential Equations.

[2] Heuer, N. and Sayas, F.-J., *Crouzeix–Raviart Boundary Elements.* Report 07/5, BICOM, Brunel University, 2007.

*This research was partially supported by Fondecyt project number 1080044.

†Department of Mathematical Sciences, Brunel University, Uxbridge UB8 3PH, UK, e-mail: martin.healey@brunel.ac.uk

‡Facultad de Matemáticas, Pontificia Universidad Católica de Chile, Casilla 306, Correo 22, Santiago, Chile, e-mail: nheuer@mat.puc.cl

Superconvergence scheme of a locking free FEM in a Timoshenko optimal control problem*

Erwin Hernández† Enrique Otárola‡

Abstract

In this work we analyze the numerical approximation of an optimal control problem of a Timoshenko beam, by considering two kind of distributed control: on the displacements and/or on the rotations. The discretization of the control variables is using piecewise constant functions. The state and the adjoint state are discretized by a locking free scheme of linear finite elements. An interpolation postprocessing technique is used to the approximations of the optimal solution of the continuous optimal control problem. It is proved that these approximations have superconvergence order h^2.

References

[1] A. Rösch and B. Vexler, *Optimal dontrol of the Stokes equations: A priori error Analysis for finite element discretization with postprocessing*, SIAM J. Numer. Anal., 44 pp. 1903–1920, (2006).

[2] Y. Chen, *Superconvergence of mixed element methods for optimal control problems*, Math. Comp., 77 , pp. 1269-1291 (2008).

[3] E. Hernández and E. Otárola, *A locking free FEM in active vibration control of a Timoshenko beam*, submitted.

[4] C. Meyer and A. Rösch, *Superconvergence Properties of Optimal Control Problems.* SIAM J. Control Optim. Volume 43, Issue 3, pp. 970-985 (2004).

*This research was partially supported by FONDECYT 1070271, BASAL projects CMM - Universidad de Chile, and DGIP - USM 120851.

†Departamento de Matemática, Universidad Técnica Federico Santa María, Casilla 110-V, Valparaíso, e-mail: erwin.hernandez@usm.cl

‡Departamento de Matemática, Universidad Técnica Federico Santa María, Casilla 110-V, Valparaíso, e-mail: enrique.otarola@usm.cl

Hybrid boundary elements scheme for modeling flat structures in $\mathbb{R}^3$

Carlos Jerez-Hanckes * Jean-Claude Nédélec †

Abstract

We present an augmented boundary element method for modeling elliptic and wave propagation problems in $\mathbb{R}^3$ with Dirichlet conditions imposed over flat surfaces having very large aspect ratios. For this, we study unbounded connected domains $\Omega := \mathbb{R}^d \setminus \bar{\Gamma}_m$, $d = 2, 3$, where Γ_m is an orientable manifold of co-dimension one, e.g., a line segment or a plane. Thus, Ω is not even Lipschitz and problems defined therein usually fall in the category of screen, crack or interface problems [1], [2], [3], for which solutions are known to possess singular behaviors [4] and classical Galerkin or collocation methods show poor convergence. The talk is organized as follows. We first analyze simple problems in $\mathbb{R}^2$ with Γ_m described by a Jordan curve, and observe that the associated single layer potentials can be reduced to compactly perturbed logarithmic integral operators. Their solutions are shown to be accurately given by weighted Tchebychev polynomials. Then, we extend these ideas to manifolds in $\mathbb{R}^3$ with only one bounded direction, e.g., infinite strips or cylinders, and show that localized single layer operators also portray logarithmic singularities. Entirely bounded surfaces with large length-to-width ratios are next considered. If the manifold boundary $\partial\Gamma_m$ is Lipschitz, corner singularities may show up. To handle this, our numerical scheme uses the previous observations and employs different discretization bases according to the encountered singular behavior – corner or edge. As a concrete application, we show results for electrostatic and elastic wave generation problems for the so-called surface acoustic waves interdigital transducers (SAW IDTs) [5].

References

[1] E. Stephan. *Boundary integral equations for screen problems in* $\mathbb{R}^3$. Integral Equations Operator Theory, vol. 10, 2, pp. 236-257, (1987).

*Seminar für Angewandte Mathematik, ETH Zürich, Switzerland, e-mail: `cjerez@math.ethz.ch`

†Centre de Mathématiques Appliquées, École Polytechnique, 91128 Palaiseau, France, e-mail: `nedelec@cmap.polytechnique.fr`

[2] M. Costabel and M. Dauge. *Crack singularities for general elliptic systems*. Mathematische Nachrichten, vol. 235, 1, pp. 29-49, (2002).

[3] S. Nicaise and A.-M. Sändig. *General interface problems* I. Mathematical Methods in Applied Sciences, vol. 17, 6, pp. 395-429, (1994).

[4] P. Grisvard. *Elliptic Problems in Nonsmooth Domains*. Pitman, Ireland, 1985.

[5] C. Jerez-Hanckes, V. Laude, J.-C. Nédelec and R. Lardat. *3-D Electrostatic Hybrid Elements Model for SAW Interdigital Transducers*. IEEE Ultrasonics, Ferroelectrics and Frequency Control Transactions vol. 55, 3, pp. 686–695, (2008).

Moving mesh finite element methods for the adaptive solution of transient PDEs with moving boundaries

PETER JIMACK *

Abstract

This talk will describe a new adaptive finite element algorithm for the solution of nonlinear diffusion equations using moving grids. The technique is particularly appropriate for problems with moving boundaries: both external boundaries, where the problem domain is time-dependent, and internal boundaries such as interfaces between phases. The approach is based upon conserving the distribution of a monitor function across the spatial domain throughout time, and this conservation principle is used to drive the velocity of the nodes in the moving mesh. A number of computational examples will be presented from a wide range of sample applications including the porous medium equation (second order nonlinear diffusion), droplet spreading problems (fourth order nonlinear diffusion) and phase-change problems. The computational accuracy and the practical efficiency of the scheme will be assessed and a number of physical properties (conservation, self-similarity, waiting times, etc.) will be discussed. This is joint work with Mike Baines (Reading/Leeds) and Matthew Hubbard (Leeds).

*School of Computing, University of Leeds, Leeds, LS2 9JT, UK, e-mail: pkj@comp.leeds.ac.uk

Estimates for Raviart-Thomas and Nédélec elements on anisotropic meshes

Ariel L. Lombardi*

Abstract

In this talk we consider estimates for the Raviart-Thomas [6, 7] and Nédélec [4] interpolation operators of any order on tetrahedral meshes with arbitrarily narrow elements. More precisely, we obtain interpolation error estimates on meshes satisfying two different geometrical restrictions, namely, the "regular vertex property" (RVP) and the "maximum angle condition" (MAC) [1]. These two conditions allow for meshes that not satisfy the standard shape regularity assumption [2], which appear naturally, for instance, in the approximations of boundary layers. The RVP is a stronger condition than the MAC. The estimates are obtained in each element of the mesh paying attention to the dependence of the constants on the geometrical properties of the element. The global estimate is obtained adding the individual ones. Then we are interested in two kind of estimates: (1) estimates valid uniformly for all elements having a particular geometrical property, and (2) anisotropic estimates also valid uniformly for some class of elements. We say that an estimate is of anisotropic type when in front each derivative appear the lengths of the element in the corresponding directions. Related results were previously obtained, for instance, in [3] for Raviart-Thomas interpolation in two dimensions or in three dimensions under the RVP, and in [5] for the Nédélec interpolation of lowest degree. For the Raviart-Thomas interpolation we obtain error estimates valid uniformly under the MAC, but anisotropic estimates can be proved only under the RVP. This is not the case for the Nédélec interpolation, for which we can obtain anisotropic error estimates also under the MAC. These results are partly joint work with Thomas Apel, Gabriel Acosta, and Ricardo G. Durán.

References

[1] Acosta, G., Durán, R.G., *The maximum angle condition for mixed and non conforming elements: Application to the Stokes equations* SIAM J. Numer. Anal. 37, 18-36, 2000.

*Universidad Nacional de General Sarmiento, Universidad de Buenos Aires and CONICET, Argentina, e-mail: aldoc7@dm.uba.ar

[2] Ciarlet, P.G., The Finite Element Method for Elliptic Problems, North Holland, 1978.

[3] Durán, R. G., Lombardi, A. L., *Error estimates for the Raviart-Thomas interpolation under the maximum angle condition*, SIAM J. Numer. Anal., 46, 1442-1453, 2008.

[4] Nédélec, J.C., *Mixed Finite Elements in* $\mathbb{R}^3$, Numer. Math. 35 (3), 315 - 341, 1980.

[5] Nicaise, S., *Edge elements on anisotropic meshes and approximation of the Maxwell equations*, SIAM J. Numer. Anal. 39 (3), pp. 784-816.

[6] Raviart, P.A., Thomas, J.M., A mixed finite element method for second order elliptic problems, *Mathematical Aspects of the Finite Element Method* (I. Galligani, E. Magenes, eds.), Lectures Notes in Math. 606, Springer Verlag, 1977.

[7] Thomas, J.M., Sur l'analyse numérique des méthodes d'éléments finis hybrides et mixtes, Thèse d'Etat, Université Pierre et Marie Curie, Paris, 1977.

Mixed FEM-BEM coupling for non-linear transmission problems with Signorini contact

MATTHIAS MAISCHAK*

Abstract

In this paper we generalize the approach in [5] and discuss an interface problem consisting of a non-linear partial differential equation in $\Omega \subset \mathbb{R}^n$ (bounded, Lipschitz, $n \geq 2$) and the Laplace equation in the unbounded exterior domain $\Omega_c := \mathbb{R}^n \backslash \bar{\Omega}$ fulfilling some radiation condition, which are coupled by transmission conditions and Signorini conditions imposed on the interface. The interior pde is discretized by a mixed formulation, whereas the exterior part of the interface problem is rewritten using a Neumann to Dirichlet mapping (NtD) given in terms of boundary integral operators. We treat the general numerical approximation of the resulting variational inequality and discuss the non-trivial discretization of the NtD mapping. Assuming some abstract approximation properties and a discrete inf-sup condition we prove existence and uniqueness and show an a-priori estimate, which generalizes the results in [5]. Choosing Raviart-Thomas elements and piecewise constants in Ω and hat functions on $\partial\Omega$ the discrete inf-sup condition is satisfied [1, 3]. We present a solver based on a modified Uzawa algorithm, reducing the solution procedure of the non-linear saddle point problem with an inequality constraint to the repeated solution of a standard non-linear saddle point problem and the solution of a variational inequality based on an elliptic operator. Finally, we present a residual based a-posteriori error estimator compatible with the Signorini condition and a corresponding adaptive scheme, see [6]. Some numerical experiments are shown which illustrate the convergence behavior of the uniform h-version with triangles and rectangles and the adaptive scheme as well as the bounded iteration numbers of the modified Uzawa algorithm, underlining the theoretical results.

References

[1] BABUŠKA, I., GATICA, G., *On the mixed finite element method with Lagrange multipliers.* Numer. Methods Partial Differential Equations, vol. 19, pp. 192–210, (2003).

*School of Information Systems, Computing & Mathematics, Brunel University, Uxbridge UB8 3PH, UK, e-mail: `Matthias.Maischak@brunel.ac.uk`

[2] Barrientos, M., Gatica, G., Maischak, M., *A-posteriori error estimates for linear exterior problems via mixed-FEM and DtN mappings*. Mathematical Modelling and Numerical Analysis, vol. 36, pp. 241-272 (2002).

[3] Gatica, G., Heuer, N., *A preconditioned MINRES method for the coupling of mixed-fem and bem for some nonlinear problems*. SIAM J. Sci. Comput., vol. 24, pp. 572-596 (2002).

[4] Gatica, G., Maischak, M., *A posteriori error estimates for the mixed finite element method with Lagrange multipliers*, Numerical Methods for Partial Differential Equations, vol. 21, pp. 421-450, (2005).

[5] Gatica, G., Maischak, M., Stephan, E.P., *On the numerical analysis of a transmission problem with Signorini contact via coupling of mixed-FEM and BEM*, M2AN(submitted).

[6] Maischak, M., *A posteriori error estimates for a mixed fem-bem coupling method with Signorini contact*, (submitted).

Error estimates for the finite volume discretization of the porous medium equation*

Iuliu S. Pop † Mauricio A. Sepúlveda ‡

Abstract

This work is motivated by a combined mixed finite element (MFE) - finite volume (FV) scheme of a two phase flow model for the heap leaching of copper ores modeled by a degenerate parabolic equation

$$\partial_t u - \nabla \cdot (\nabla \beta(u) + F(u)) = r(u), \quad \text{in } Q_T \equiv (0,T) \times \Omega.$$

Initially we have $u(0) = u^0$ in Ω, whereas $u = 0$ on $\partial\Omega$. In the above $0 < T < \infty$ is fixed, Ω is a bounded domain in $\mathbb{R}^d (d \geq 1)$ with a Lipschitz continuous boundary. The function $\beta : \mathbb{R} \to \mathbb{R}$ is non-decreasing and differentiable. By degeneracy we mean a vanishing diffusion, namely $\beta'(u) = 0$ for some u. We prove error estimates for the finite volume discretization for this model. Several numerical results illustrating the performance of the algorithm are provided.

References

[1] H.W. Alt, S. Luckhaus, *Quasilinear elliptic-parabolic differential equations*, Math. Z. 183 (1983) 311-341.

[2] E. Cariaga, F. Concha, M. Sepúlveda, *Flow through porous media with applications to heap leaching of copper ores*, Chemical Engineering Journal 111 (2005) 151-165.

[3] E. Cariaga, F. Concha, M. Sepúlveda, *Convergence of a MFE-FV method for two phase flow with applications to heap leaching of copper ores*, Comput. Methods Appl. Mech. Engrg. 196 (2007) 2541-2554.

*This research was partially supported by FONDAP and BASAL projects CMM, Universidad de Chile, and by Centro de Investigación en Ingeniería Matemática (CI^{2}MA), Universidad de Concepción.

†Department of Mathematics and Computer Science, Eindhoven University of Technology, P. O. Box 513, 5600 MB Eindhoven, The Netherlands, e-mail: I.Pop@tue.nl

‡CI^{2}MA and Departamento de Ingeniería Matemática, Facultad de Ciencias Físicas y Matemáticas, Universidad de Concepción, Casilla 160-C, Concepción, Chile, e-mail: mauricio@ing-mat.udec.cl

[4] R. Eymard, T. Gallouët, R. Herbin, *Finite volume methods*, in *Handbook of numerical analysis*, **Vol. VII** North-Holland, Amsterdam, 2000, 713–1020.

[5] I.S. Pop, *Regularization Methods in the Numerical Analysis of Some Degenerate Parabolic Equations*, PhD Thesis, Universitatea "Babeş-Bolyai" Cluj-Napoca, Romania, (1998).

[6] I.-S. Pop, F. Radu, M. Sepúlveda and O. Vera Villagran, *Error estimates for the finite volume discretization for the porous medium equation,* CASA Report # 13, Eindhoven University of Technology, 2008. Submitted.

Explicit solutions to nonlinear partial differential equations via nonlocal symmetries*

Enrique G. Reyes†

Abstract

In this talk I review some recent work on the theory and application of nonlocal symmetries of nonlinear PDEs. This theory was originally developed by A. Vinogradov and J. Kasil'shchik in the 1980's and 1990's as a chapter of their formal theory of differential equations. I would like to show how this theory can be implemented in such a way as to make its application straightforward and useful for applied and numerical mathematics. As examples, I present non-trivial explicit solutions (which could be used to test numerical methods) to the Kaup-Kupershmidt equation

$$q_t = q_{xxxxx} + 5\,q\,q_{xxx} + \frac{25}{2}\,q_x q_{xx} + 5\,q^2 q_x \tag{1}$$

and also a Darboux transformation for the important Camassa-Holm equation

$$2\,u_x\,u_{xx} + u\,u_{xxx} = u_t - u_{xxt} + 3\,u_x\,u\ . \tag{2}$$

References

[1] Hernandez-Heredero, R., Reyes, E.G., *Remarks on the geometric integrability of the Camassa-Holm equation.* Submitted, October 2008.

[2] Reyes, E.G., Sanchez, G., *Explicit solutions to the Kaup-Kupershmidt equation via nonlocal symmetries.* International Journal of Bifurcation and Chaos 17, 2749– 2763, (2007).

[3] Reyes, E.G., *Nonlocal symmetries and the Kaup-Kupershmidt equation.* Journal of Mathematical Physics 46 no. 7, 073507, 19 pp., (2005).

*Research partially supported by the Fondo Nacional de Desarrollo Científico y Tecnológico (FONDECYT) grant #1070191, and by the Research Ring "Anillo Ecuaciones Asociadas a Reticulados" financed by the World Bank through the Programa Bicentenario de Ciencia y Tecnología.

†Departamento de Matemática y Ciencia de la Computación, Universidad de Santiago de Chile, Casilla 307 Correo 2, Santiago, Chile, e-mail: ereyes@fermat.usach.cl

BEM–FEM coupling: back to the beginning*

FRANCISCO–JAVIER SAYAS †

Abstract

The first coupled method of finite and boundary elements originated in the late seventies and has been commonly referred to as the Johnson–Nédélec, one–equation or unsymmetric coupling. Its main drawback, as originally perceived, was the need for a particular boundary integral operator to be compact. This fact demanded smooth enough coupling interfaces (which was a clear inconvenience from the FEM point of view) and precluded its use for linear elasticity. Although it was not recognized at the time, the problem was purely theoretical in nature. We prove here that by recasting the discrete equations as a non–standard transmission problem, the lost ellipticity is recovered and that Johnson–Nédélec's coupling is stable for any pair of discrete BEM–FEM spaces.

References

[1] JOHNSON, C. AND NÉDÉLEC, J.C., *On the coupling of boundary integral and finite element methods.* Mathematics of Computation, vol 35, pp. 1063-1079 (1980).

[2] SAYAS, F.J., *The validity of Johnson–Nédélec's BEM–FEM coupling on polygonal interfaces.* SIAM Journal on Numerical Analysis, in revision.

*Research partially funded by MEC/FEDER Project MTM2007–63204 and Gobierno de Aragón (Grupo PDIE). This work was developed while the author was at the University of Minnesota supported by a Spanish MEC grant PR2007–0016.

†Dep. Matemática Aplicada, Universidad de Zaragoza (Spain) & School of Mathematics, University of Minnesota (USA), e–mail: sayas002@umn.edu

Efficient meshless methods for pseudodifferential equations on the sphere

THANH TRAN[*] DUONG PHAM[*]

Abstract

Radial basis functions are used to approximate the solutions of pseudodifferential equations on the sphere. These equations arise for example in geodesy and earth science. The use of radial basis functions ameliorates the situation when given facts are obtained as scattered data. A unified analysis for both the Galerkin and collocation methods will be discussed. Numerical experiments on relatively large scattered data point sets taken from MAGSAT satellite data will be presented.

References

[1] T. M. MORTON AND M. NEAMTU, Error bounds for solving pseudodifferential equations on spheres. *J. Approx. Theory*, **114** (2002), 242–268.

[2] T. D. PHAM AND T. TRAN, Collocation solutions to pseudodifferential equations of negative orders on the sphere using spherical radial basis functions. *Bull. Aust. Math. Soc.*, (to appear).

[3] T. D. PHAM, T. TRAN, AND Q. T. LE GIA, Numerical solutions to a boundary-integral equation with spherical radial basis functions. In *Proceedings of the 14th Biennial Computational Techniques and Applications Conference, CTAC-2008*, G. N. Mercer and A. J. Roberts, editors, volume 50 of *ANZIAM J.*, pages C266–C281, Nov. 2008. http://anziamj.austms.org.au/ojs/index.php/ANZIAMJ/article/view/1464 [November 10, 2008].

[4] T. TRAN AND T. D. PHAM, *Pseudodifferential equations on the sphere with radial basis functions: Error analysis*, 2008. Submitted to SIAM J. Numer. Anal., http://www.maths.unsw.edu.au/applied/pubs/apppreprints2008.html.

[*]School of Mathematics and Statistics, The University of New South Wales, Sydney 2052, Australia, e-mail: thanh.tran@unsw.edu.au

A parameter–uniform finite difference method for a singularly perturbed multiscale linear dynamical system*

S. Valarmathi † John J.H. Miller ‡

Abstract

A system of singularly perturbed ordinary differential equations of first order with given initial conditions is considered. The leading term of each equation is multiplied by a small positive parameter, which can be arbitrarily small. These parameters are, in general, unequal. The components of the solution exhibit overlapping layers corresponding to the various distinct time scales occurring in the solution of the problem. It is well-known that standard numerical methods do not perform in a robust way, when they are used to solve a singularly perturbed problem of this kind. In this talk a new numerical method is constructed. First a Shishkin piecewise–uniform mesh is introduced, which is used, in conjunction with a classical finite difference discretisation, to form the new numerical method for solving this problem. It is then proved that the numerical approximations obtained from this method are essentially first order convergent uniformly with respect to all of the parameters. Extensive numerical computations are presented to illustrate the utility of this new method in practice.

References

[1] P.A. Farrell, A. Hegarty, J. J. H. Miller, E. O'Riordan, G. I. Shishkin, *Robust Computational Techniques for Boundary Layers*, Applied Mathematics & Mathematical Computation (Eds. R. J. Knops & K. W. Morton), Chapman & Hall/CRC Press (2000).

[2] S.Hemavathi, S. Valarmathi, *A parameter uniform numerical method for a system of singularly perturbed ordinary differential equations*, Proceedings of the International Conference on Boundary and Interior Layers, BAIL 2006, Goettingen (2006).

*This research was supported by INCA. The first author was also supported by the UGC, New Delhi, India under the Minor Research Project-X Plan period

†Department of Mathematics, Bishop Heber College(Autonomous), Tiruchirappalli-620 017, Tamil Nadu, India, e-mail: valarmathi07@gmail.com

‡Institute for Numerical Computation and Analysis, Dublin, Ireland, e-mail: jm@incaireland.org

[3] J. J. H. Miller, E. O'Riordan, G.I. Shishkin, *Fitted Numerical Methods for Singular Perturbation Problems*, World Scientific Publishing Co., Singapore, New Jersey, London, Hong Kong (1996).